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Calculational versus mechanistic mathematics in propelling the development of physical knowledge

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A design experiment with a manipulation of instructional conditions across two groups of students was conducted to investigate the role of mathematics in solving problems involving a physical system. The instruction utilized a model-eliciting activity in the context of controlling robot movements. One group was encouraged to use mathematics as a *calculational* resource for being precise about numerical operations for transforming input values into desired output values. In contrast, the second group was encouraged to use mathematics as a *mechanistic* resource for describing their intuitive ideas about the physical quantities and their relationships. Both groups engaged in high levels of productive mathematical engagement, inventing, justifying, and evaluating valid strategies for controlling robot movements using mathematics. But only the *mechanistic* group made significant learning gains and were more likely to use their ideas on a transfer design task. Examples of the invented strategies and the talk about those strategies in whole class discussions suggest the students in the contrasting orientations thought about them in substantively different ways leading to differences in learning.

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In physical systems multiple features must be attended to and coordinated to predict an outcome. Tools that organize thinking for learners can improve their understanding. Schwartz, Martin, and Pfaffman (2005) showed how mathematics can be such a tool by explicitly prompting middle-school-aged students to use math in the balance scale task. The students were more likely than their age-group norms to use the two key features—weight and distance—simultaneously. Despite significant development gains, the majority of participants did not reach the highest level of reasoning. Mathematics helped consider possible alternative structures for coordinating features, but did not provide a basis for choosing between those alternatives beyond testing out each empirically.

Using mathematics in more principled ways may provide an additional benefit. In a flood prediction task, Kaplan and Black (2003) provided middle-school-aged students cues about the mechanisms by which each feature may impact water levels. The mechanistic cues caused students to engage in more mental animations of the system, which led to more focused investigations of causal effects of individual features and better predictive accuracy during those investigations. Using mathematics specifically to represent mechanisms may facilitate development as students can rule out many feature effects and interactions using internal animations and focus on testing only plausible ones.

The present study used a physical system context in which students were likely to have intuitions about mechanisms that relate system features—middle school students learning to program simple robot movements. Students have intuitive ideas about how wheel rotations and wheel size relate mechanistically to produce movement distances, but these ideas are rarely fully articulated. Thus, we were able to investigate different orientations for math use: (1) a *mechanistic* orientation in which math is used as a tool for modeling physical intuitions about the way the system works; versus (2) a *calculational* orientation in which math is used as a tool for describing input-output patterns induced empirically. We used a teaching experiment design in order to investigate the development in reasoning and also the interactions between students as they communicated their ideas.

Instructional Design

We developed the *Robot Synchronized Dancing* (RSD) instructional unit (Silk, Higashi, Shoop, & Schunn, 2010), in which students program multiple LEGO robots to dance in sync with each other in a model-eliciting activity (Lesh, Hoover, Hole, Kelly, & Post, 2000). Students work in teams of 2-3 in a series of express-test-revise cycles to invent solutions, accounting for and coordinating the proportional relationships (Lamon, 2007) between features of the robots' physical design, the program parameters, and the magnitude of the robots' movements.

Methods

Eighteen students from an independent middle school participated. They were split in two sections, each of which met five consecutive days, 2.5 hours per day at a university research building. The sections were assigned randomly to conditions—*Mechanistic* (n=10) or *Calculational* (n=8). Students chose their section based on convenience, but were not informed of the differences between sections. The first author was the instructor for both sections. The unit was implemented similarly between sections, except for three distinctions intended to activate the contrasting mathematics orientations:

(1) *Design task setup* – each cycle was introduced to the students as a design task, with the *Mechanistic* group asked to represent their intuitions about how the robots work and the *Calculational* group asked to generate steps for getting desired outcome values from input values;

(2) *Instructional support* – the questions used by the instructor while students invented their strategies were focused on connecting quantities and operations to the physical situation in the *Mechanistic* group and focused on correctness of calculations in the *Calculational* group; and

(3) *Teacher-provided cases* – after inventing their own strategies, students analyzed example strategies that illustrated key understandings, with cases given to the *Mechanistic* group

focused on identifying and incorporating key intermediate physical quantities and cases given to the *Calculational* group focused on identifying empirical patterns and numerical operations that generate the patterns.

Data sources included video records of the RSD tasks and a post-instruction competition design task, posters of teams' invented strategies and other written work, team post-interviews, and 12-item pre- and post-assessments.

Results

Analyses of the whole class discussion posters and talk suggest that math was central to the activity of both groups, but they connected math to the situation in substantively different ways. All posters from both groups included explicit numerical operations. Whereas some *Mechanistic* group posters included robot physical parameters explicitly (5/15), none of the *Calculational* group posters did (0/15). The *Calculational* group discussions did include high-level talk about mathematics—connecting math ideas to the situation and building off each other's ideas to find more explicit, general solutions—but unlike the *Mechanistic* group they did not use mental animations or physical mechanisms. The *Calculational* group posters were more likely to include an under-specified solution (i.e., requiring guess and test), and generated more complex numerical solutions to accommodate this. Rich examples and more details about the contrasting solutions and talk will be a focus of the presentation.

In post-interviews, all four *Mechanistic* teams but only one *Calculational* team reported using the invented RSD strategies in the design competition task. Inspecting the assessment, *Mechanistic* group students made significant learning gains [$M_{pre}(SD)=0.49(0.27)$, $M_{post}(SD)=0.71(0.22)$, $t(9)=3.34$, $p=0.008$, $d=0.80$], whereas *Calculational* group students did not [$M_{pre}(SD)=0.50(0.17)$, $M_{post}(SD)=0.60(0.24)$, $t(7)=1.67$, $p=0.14$, $d=0.63$]. This supported the hypothesis that a mechanistic orientation has an additional benefit when using math for understanding, although both groups had somewhat large effect sizes.

Conclusion

In both conditions, students used and reasoned about mathematics in sensible ways. As a result, both invented valid strategies, which is consistent with prior research on development of physical knowledge as a result of using mathematics. In addition, setting up the math use in a mechanistic orientation led to simpler, more focused ideas and problem solving strategies grounded in the situation, in turn leading to significant learning gains. This study provides clarification on how the benefit for using mathematics for developing conceptual understanding may be about using simpler mathematics (Iversen & Larson, 2006) focused on representing ideas about the way systems work.

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