Calculational versus **mechanistic** mathematics in propelling the development of physical knowledge

Eli M. Silk and Christian D. Schunn
University of Pittsburgh

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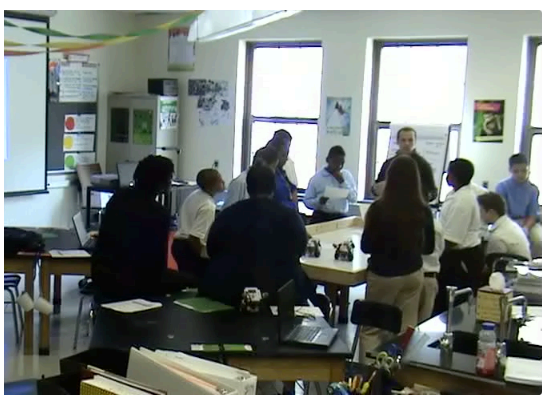
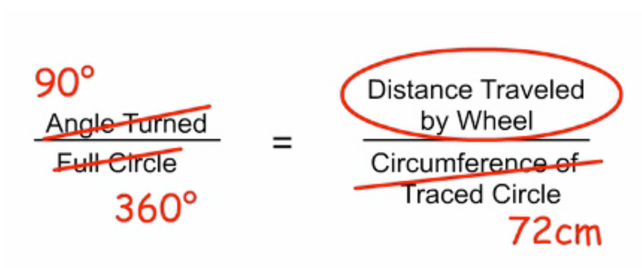
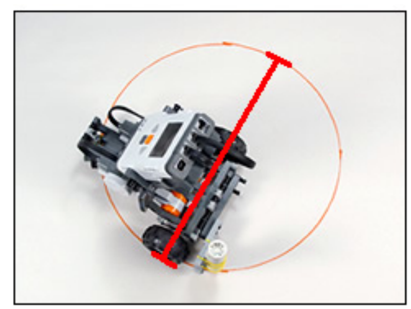






“How Mathematics Propels the Development of Physical Knowledge”

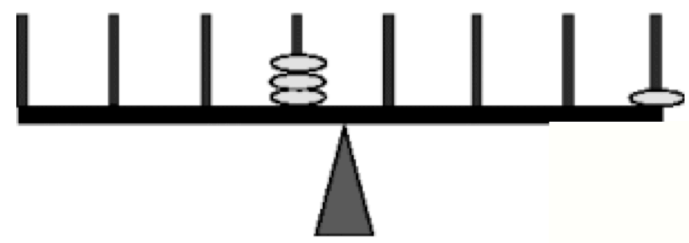
(Schwartz et al., 2005) – Which side will fall?

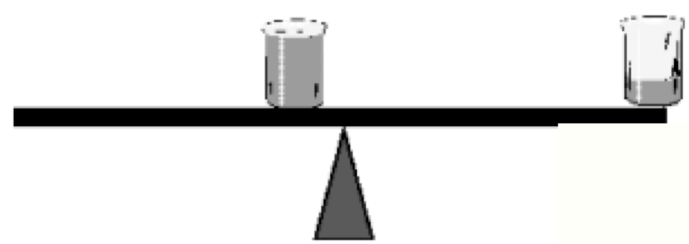
- Hard-to-measure quantities
  (vs discrete quantities)
  - 10-yr-olds = 5yr-olds
  - Focus solely on weight (Ignore distance)
- “Show your math”
  (vs “Explain your answer”)
  - 11-yr-olds = Adults
  - Use weight and distance simultaneously
- Math helps organize thinking
  - *Both **quantities** and **operations***
  - *But limited in helping to choose between alternatives (need empirical testing)*
- Thinking about **MECHANISMS** can (Kaplan & Black, 2003)
  - Mental cues helps students engage in mental animations
  - Leads to more focused investigations of causal effects and better predictive accuracy in those investigations

Moment = Force X Distance

3 x 1 ? 1 x 4

3 < 4



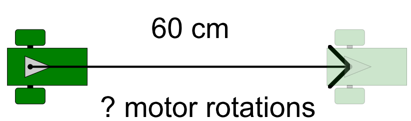


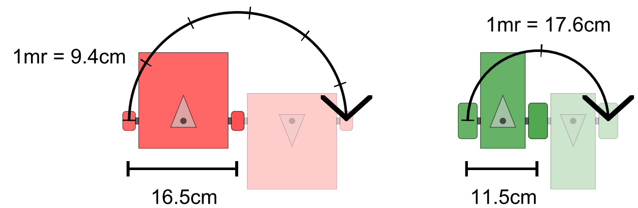
Context for investigating coordination of math and mechanisms?

Controlling Robot Movements

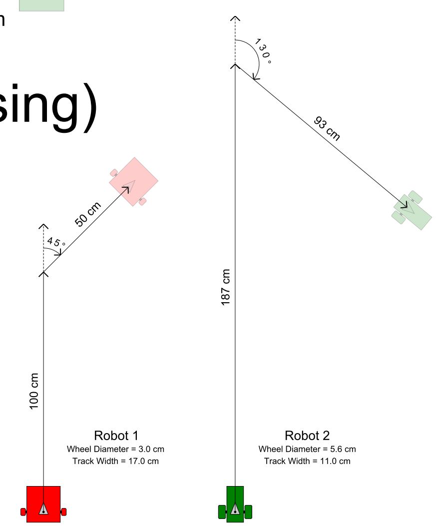
- **Patterns/relationships** are inspectable, manipulable, & reliable
  - Good for learning how students incorporate **MATH** and **MECHANISMS**
  - Robot Movements ↔ Program Parameters ↔ Physical Features

Distance = Motor Rotations × Wheel Circumference



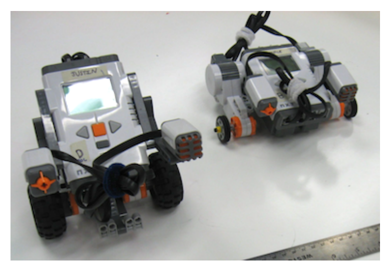


- Engaging BUT lends itself to **playing around** (guessing)



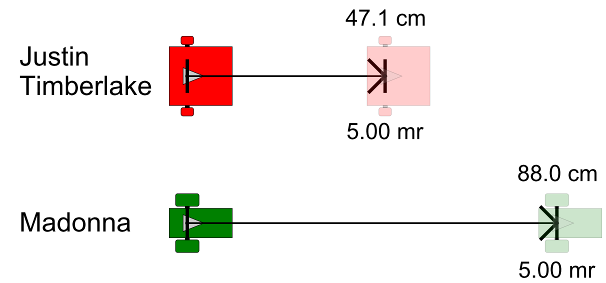
ROBOT SYNCHRONIZED DANCING

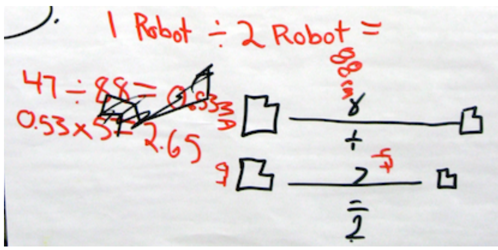
- Develop a "toolkit" for a dance team captain
- Model-Eliciting Activity (MEA) (Lesh et al., 2000)



Contrasting Math-To-Robot Approaches

Rotations →
[Wheels →]
Distance





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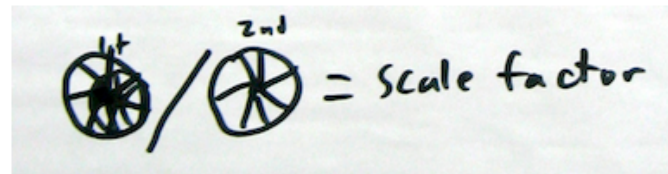

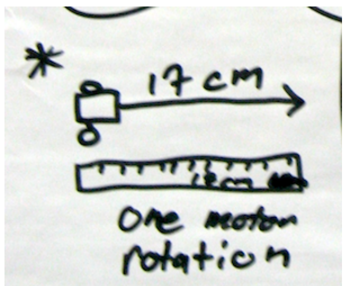

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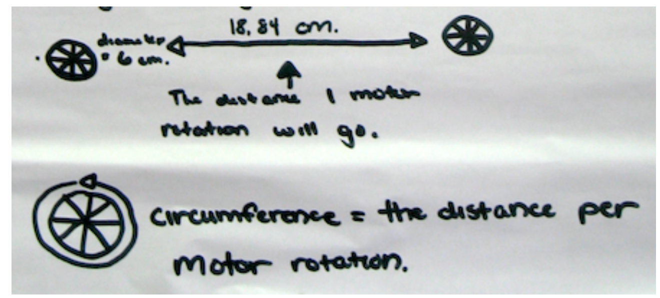


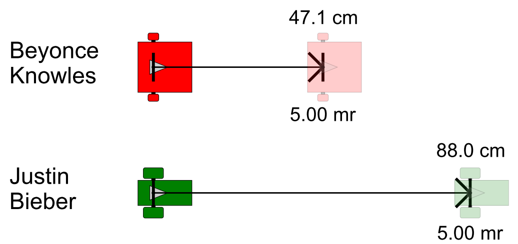
CALCULATIONAL
(Thompson et al., 1994)

MECHANISTIC
(Kaplan & Black, 2003; Russ et al., 2008)

Our claim – math-to-robot approaches w/ vs w/o explicit mechanisms are **numerically** the same (use the same mathematical understanding resources), but **cognitively** different (use different physical understanding resources), so will support different learning

Study Design

Do different instructional framings of the use of mathematical resources lead to different understandings?



- Research setting 1-week in summer

- Participants – 2 Groups
  - Students assigned based on time availability, but groups randomly assigned to condition
  - 5^{th}-7^{th} grades (16/18 in 5^{th} or 6^{th})
  - *Mechanistic* (n=10)
  - *Calculational* (n=8)

- Student Work (Posters, Discussions)
- Pre/Post Assessment (10-items)
- Post-Instruction Competition Task

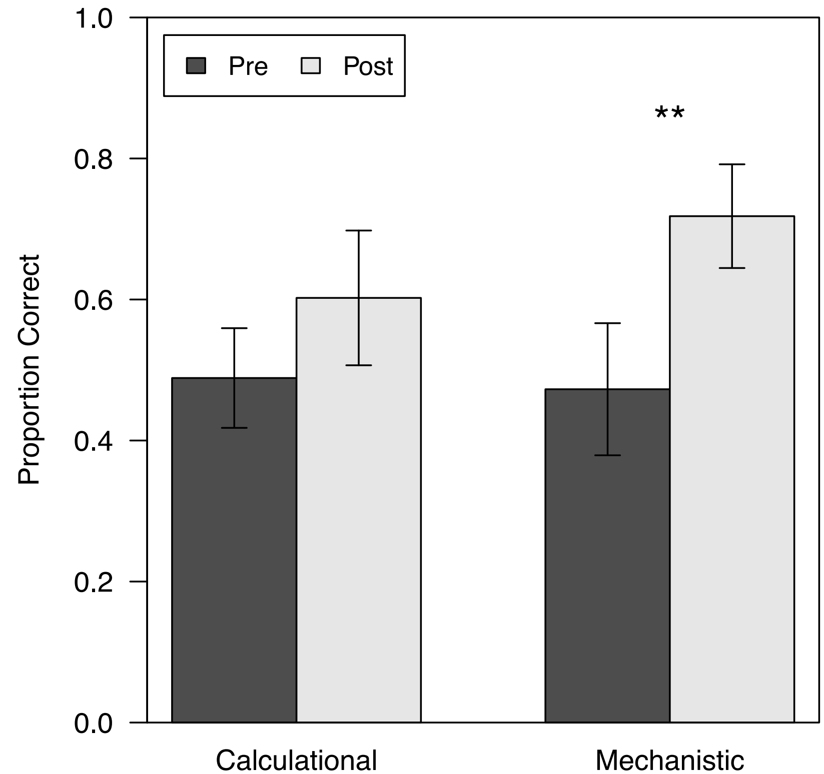
- Mechanistic vs Calculational (Contrasting Instructional Resources and Framings)
  - Design Task Setup
    - Modeling intuitions (mechanistic) versus input-output focus (calculational)
  - Teacher Cases
    - Identifying role of physical features (mechanistic) versus identifying empirical patterns (calculational)
  - Instructional Support
    - Focus on explaining what quantities mean (mechanistic) versus on seeing numerical patterns in data (calculational)

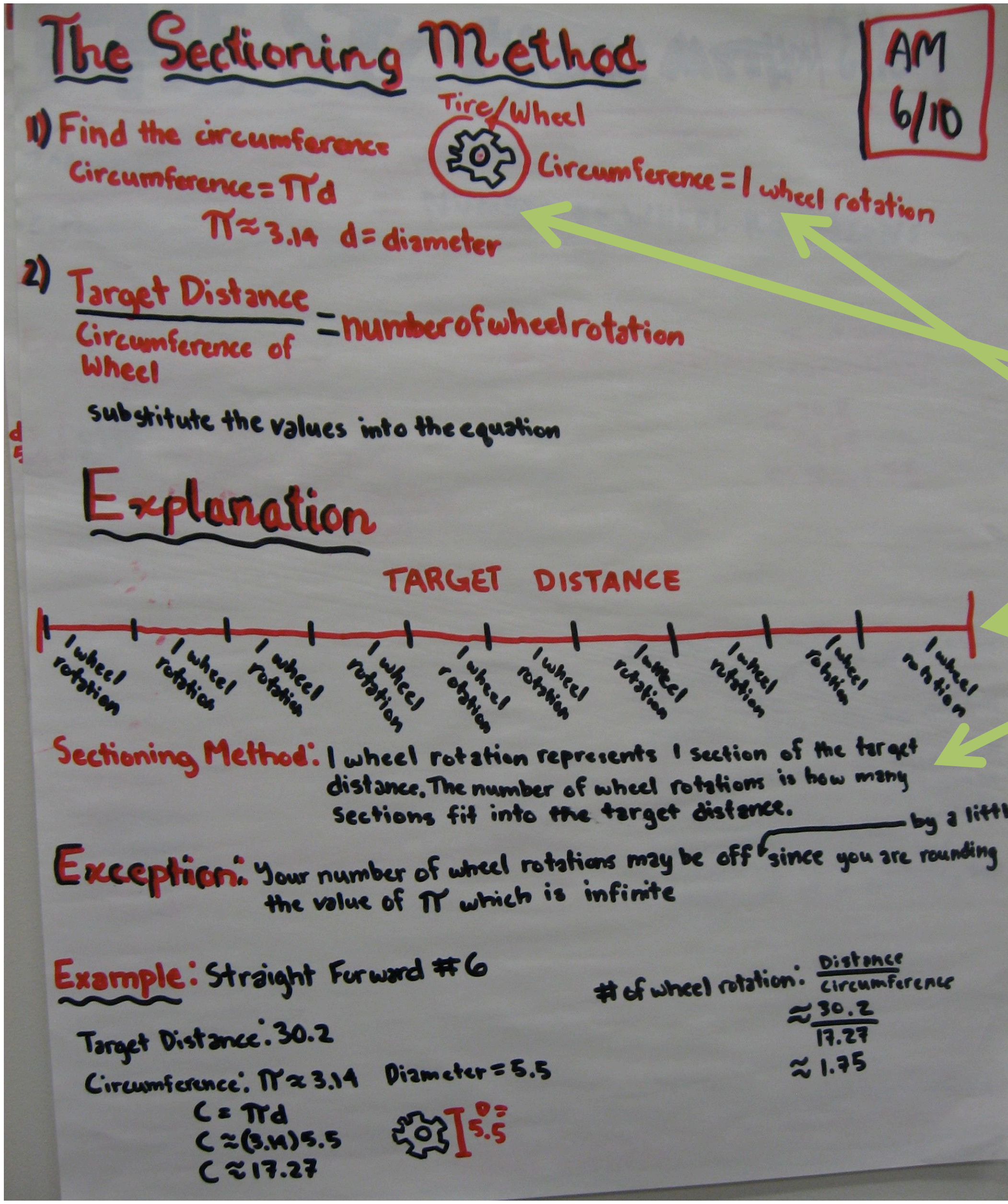
Pre-Post Test Results

- Repeated Measures ANOVA suggests significant main effect of time (Pre-Post)
  - $F(1,16) = 11.05$, $p < .01$

- Follow-up tests suggest that only the ***Mechanistic*** Group reliably improves Pre-Post
  - Mechanistic Group
    Gain = .23, 95% CI [.09, .37]
  - Calculational Group
    Gain = .10, 95% CI [-0.06, .26]

- **What about their work?**

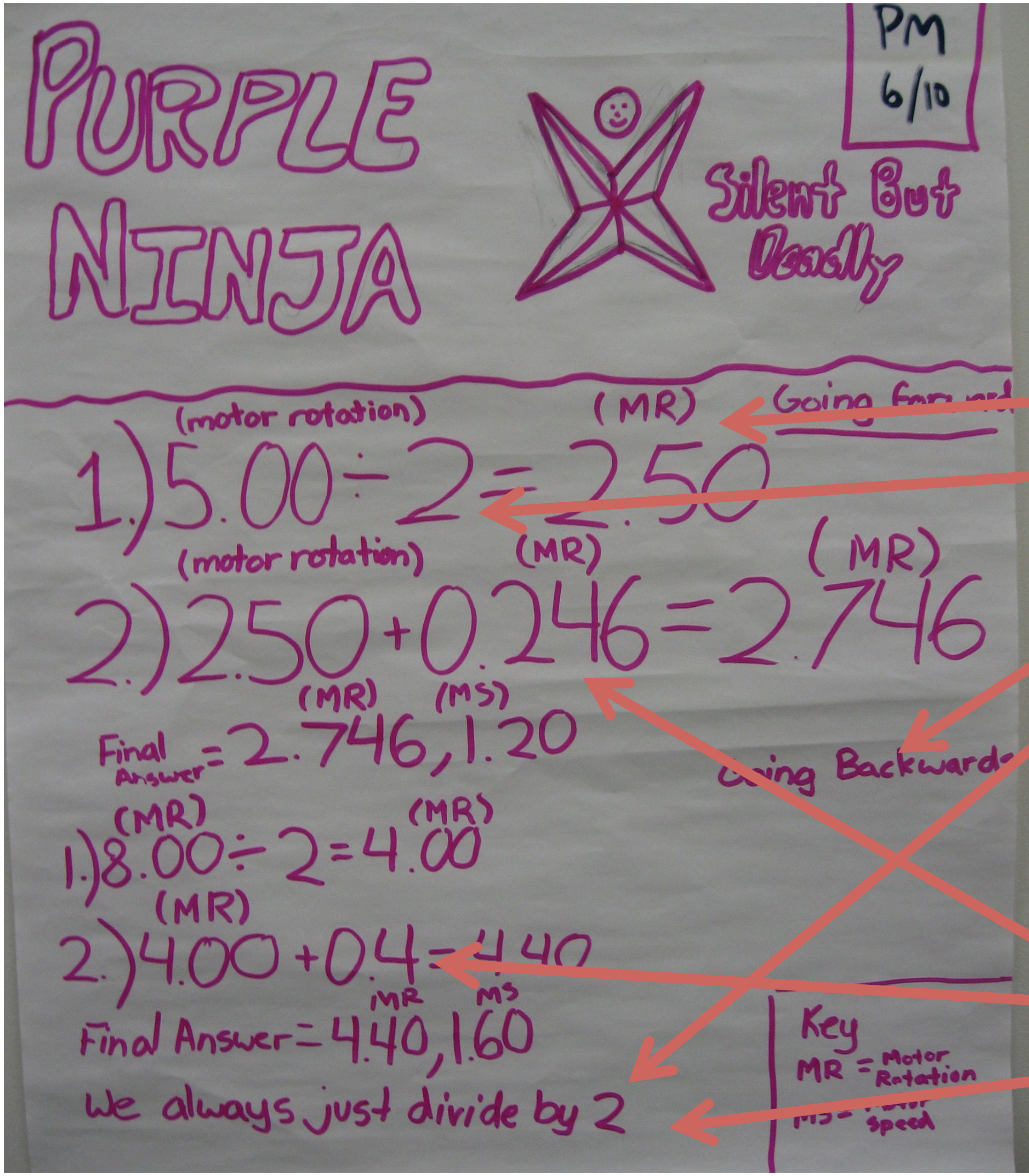




Poster Analysis

High Mechanistic

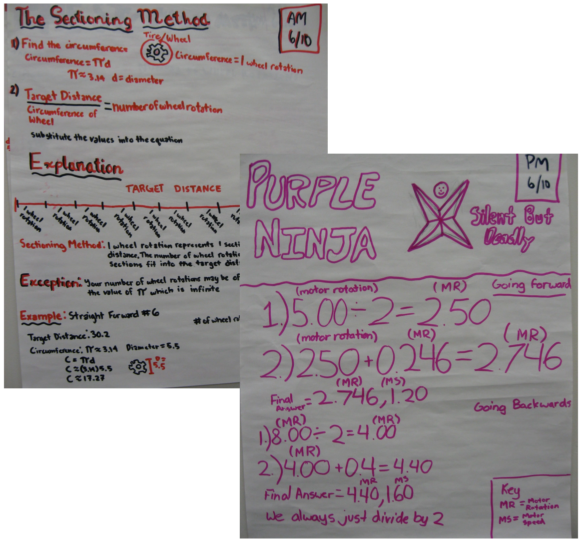
- Mechanistic Score
  - ✓ Physical Features
  - ✓ Label Intermediate Values
  - ✓ Situation Pictures
  - ✓ Explanation
- Quality Score
  - ✓ Steps Clear
  - ✓ Valid
  - ✓ Fully-Specified
  - ✓ Generalized



Poster Analysis

Low Mechanistic

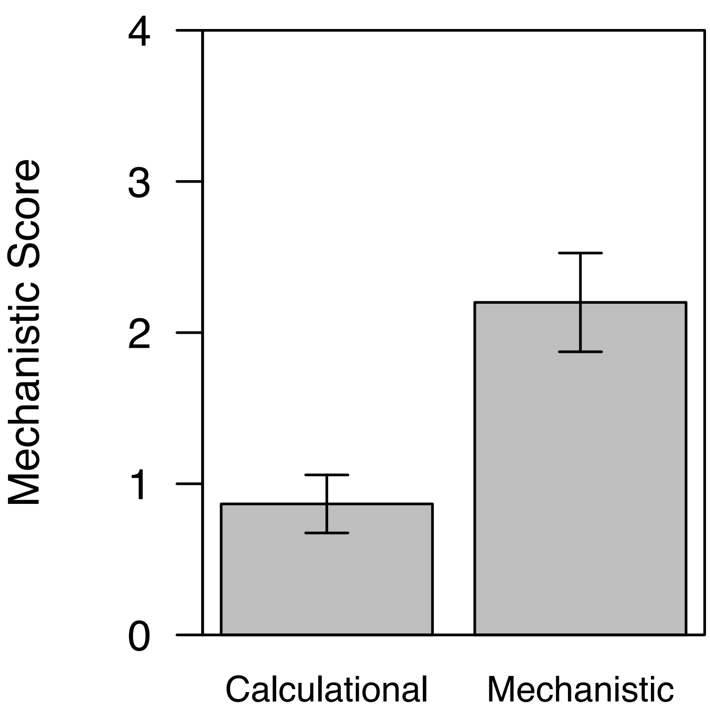
- Mechanistic Score
  - ✗ Physical Features
  - ✗ Label Intermediate Values
  - ✗ Situation Pictures
  - ✗ Explanation
- Quality Score
  - ✓ Clear Steps
  - ✓ Valid
  - ✗ Fully-Specified
  - ✗ Generalized



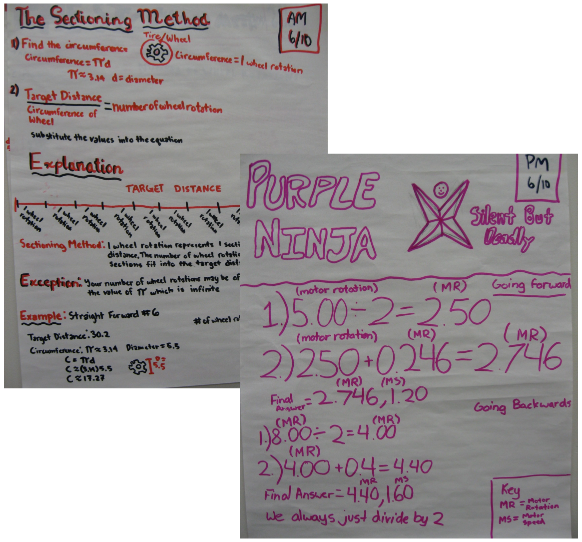
Does the Mechanistic group **think** about the task differently?

Poster Mechanistic Score

# Posters with the feature (out of 15)	Calculational	Mechanistic
Physical Features	0	6
Label Interm. Values	8	12
Situation Pictures	1	7
Explanation	4	8



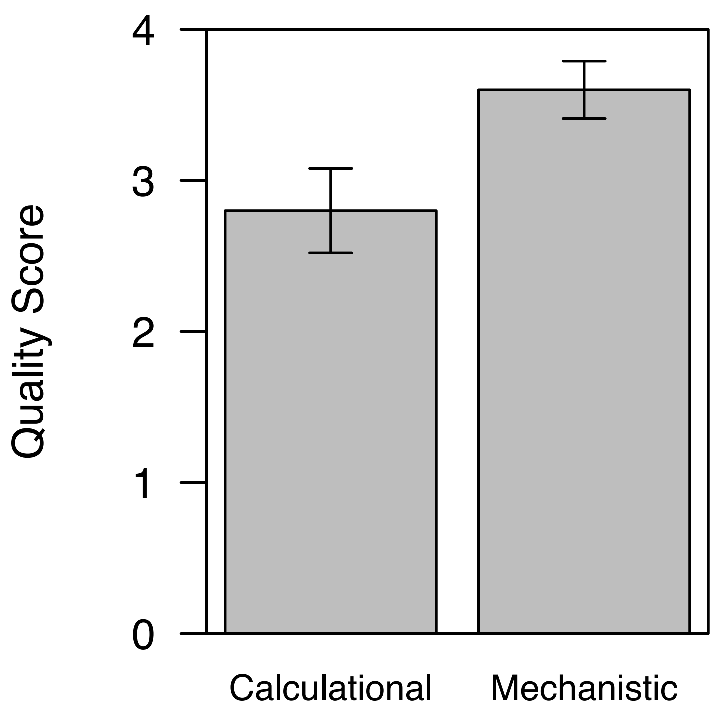
- **YES**, manipulation worked well
  - Based solutions on physical features
  - Used images (not just numbers/operations)
- Mechanistic thinking **not easy**
  - **Not ALL** *Mechanistic* teams adopted it
  - But **No** *Calculational* teams did



Does the Mechanistic group invent better **solutions**?

Poster Quality Score

# Posters with the feature (out of 15)	Calculational	Mechanistic
Valid	13	13
Clear Steps	15	15
Fully Specified	6	15
Generalized	8	11

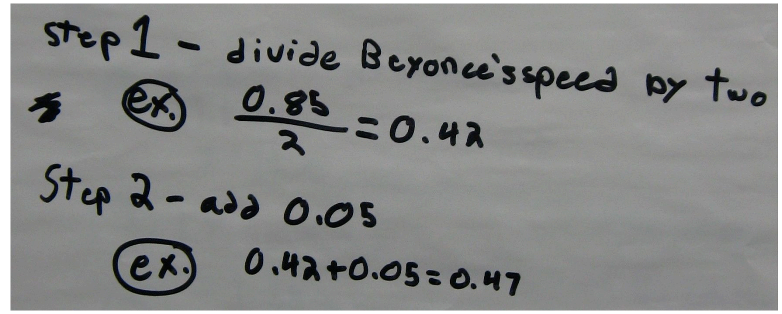


- **SORT OF**, no differences in some ways
  - Both invent strategies that work (valid)
  - Both articulate strategies well
- Important differences in other ways
  - Less reliance on adjusting or guessing
  - More generalizing beyond current context

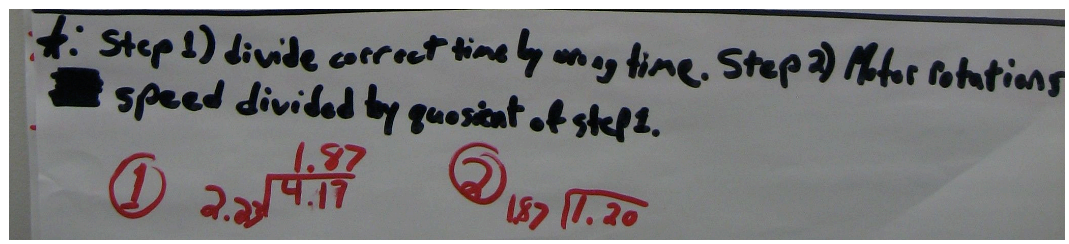
Do the Calculational teams just do low-level math?
(procedures without connections)

NO!!

- They do connect their math to the situation (in terms of inputs & outputs)
  - “Since Beyonce’s always half as slow as Justin, we decrease Justin’s speed by half”



- They do make connections to and build off each other’s ideas
  - “It’s showing the, um, like how, sort of like how the Green team had divided by two, but we wanted it more exact number ... the more exact number of how much the time, of how much the speed is. It’s a bit less than half the time.”

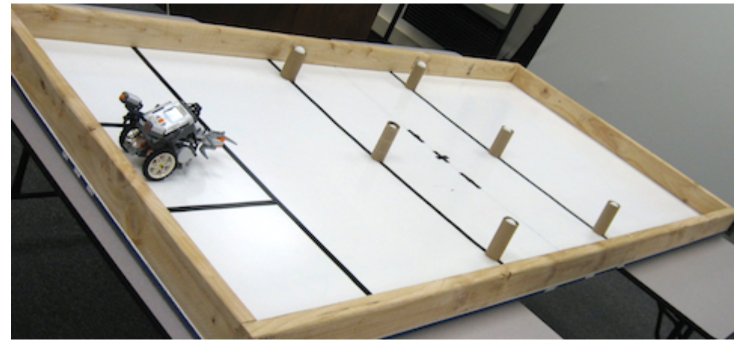


- They do recognize when they don’t have a solution or explanation
  - the “Feeling” strategy & “that’s too smart”

- **Why? They are limited by focusing only on their mathematical resources**
  - Don’t use physical features or mental animations/images to evaluate their mathematical choices

Transfer Competition Task

Did you use any of the strategies from this week?



Mechanistic (4/4 teams)

- Purple Team
  - S1: We used the, the strategies that we learned all throughout the week. Um, we, like, for the straights, we, um, used the circumference of the wheel as the rotations and measured it, measured the area.
  - I: What do you mean by measured the area?
  - S2: Like how far it was from here to here. And then we like said, I think the wheel was 26 cm, so we said one rotation would be 26 cm, two would be whatever that is times two.

Mechanistic teams see the underlying similarities between the problems

Calculational teams see this as a new problem (different robot, not comparing)

Calculational (1/4 teams)

- Red Team
  - S: "Not really. No. Cause there isn't any, like, it isn't like we are comparing two different robots to do the same thing. All robots are the same in this ... So there really is no need for any strategies like that."
- Purple Team
  - S1: "Cause it's a different robot. It has bigger wheels."
  - S2: "Well, we don't know like, I don't really know why we didn't use one of our strategies. We just decided to use one and didn't really think about the others."
  - S1: "We're still in the lead."
  - I: "So it's working for you?"
  - S1, S2: "Yeah"

Summary

- The two groups approached the task in substantively different ways
  - Representing images/animations of mechanisms versus capturing numerical patterns
  - But both did engage in productive mathematics and sense-making
- The Mechanistic group
  - learned more,
  - had higher quality strategies, and
  - more likely to use those strategies in a transfer competition task

Potential Significance

- Math can be a real tool for situational understanding
  - Students have different types of **cognitive resources** available to them
    - mathematical and physical
  - The **framing** of problems make those resources more or less **accessible**
    - available and salient
  - Mathematical resources can serve to “organize” thinking, but physical resources (mechanisms) can serve to “focus” thinking
    - they are mutually supportive and together are powerful

Thank You

Eli M. Silk
esilk@pitt.edu