Resources for Learning Robots

Facilitating the Incorporation of Mathematical Models in Students' Engineering Design Strategies

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Abstract: Two design experiments were conducted to investigate students' use of mathematics in their engineering design strategies. The instruction utilized a model-eliciting activity in the context of controlling robot movements. Analyses of pre-post problem solving assessments from the first study support the hypothesis that instruction does facilitate improved understanding of how the robots move. These same gains are not evident in students who work with the same robots in preparation activities for robot competitions. Cases of student teams that developed more and less sophisticated design strategies suggested students' epistemological orientation explains some of the observed differences. The second study then manipulated epistemological orientation and results support the hypothesis that a mechanistic orientation results in greater learning gains.

Objectives

The larger goal of this research is to understand how students can become more capable engineering designers by better understanding the physical systems within which they work. The objective of this current study was to investigate the role of mathematics in facilitating that understanding.

Theoretical Framework

Mathematical modeling is a key part of authentic engineering as practitioners must work with complex physical systems (Gainsburg, 2006). Beyond modeling, using mathematics can facilitate understanding of the physical system itself (Schwartz, Martin, & Pfaffman, 2005). Mathematical procedures learned without connections to the physical quantities and relations that they represent are unlikely to lead to conceptual understanding. Nevertheless, qualitative understanding alone is not sufficient for complex problems, so connecting qualitative understanding with mathematics is ideal (Lehrer & Schauble, 1998).

For simpler and familiar problems, informal strategies are better tied to the situation and so are less error prone than abstract strategies (Koedinger, Alibali, & Nathan, 2008). Guess-and-check is one example of a grounded, informal strategy (Nhouyvanisvong, 1999). Students who use guess-and-check can be systematic and purposeful (Johanning, 2004; Levin, 2008), using implicit knowledge about the situation's functional relationships. Nevertheless, there are levels of guess-and-check (Stacey & MacGregor, 1999), but guess-and-check is not easily extended to complex cases. It is also unclear how students can transition from such strategies to models that are explicit about the relevant quantities and relations. Investigating contexts where students make such transitions could provide insight into students' progressions in the use of mathematics as a tool for understanding.

As a framework for this study, we adopted a resources view of knowledge (Hammer, Elby, Scherr, & Redish, 2005; Smith, diSessa, & Roschelle, 1993). We assumed students have a complex system of context-sensitive knowledge elements, including many intuitive ideas about the physical world. These knowledge elements can be utilized as productive resources for building more refined and organized understanding. To that end, we seek to identify knowledge elements that students draw on when solving engineering problems using mathematics, and to highlight productive paths for refining and reorganizing those elements. Our research questions:

1. When working in an engineering domain that readily involves guess-and-check, how can instruction facilitate students in using more explicit mathematical models?
2. Does using more explicit mathematical models increase students' understanding of how the robots work?
3. How can the instructional environment better facilitate students' use of mathematics to identify and build on their existing cognitive resources?

Instructional Design

Robotics is an interesting case of engineering design that is becoming increasingly popular with K-12 student competitions (e.g., FIRST – For Inspiration and Recognition of Science and Technology). But engineering in robot competitions may not lead to understanding how the robots work more generally. Teams may develop solutions fine-tuned for a particular challenge without finding a need to understand how the general system works.

As an alternative, we developed the *Robot Synchronized Dancing* (RSD) instructional unit (Silk, Higashi, Shoop, & Schunn, 2010). The RSD unit facilitates students in programming multiple LEGO robots to dance in sync with each other. To investigate students' use of mathematics, the unit was designed as a model-eliciting activity (MEA) in which students invent solutions in a series of express-test-revise cycles (Hamilton, Lesh, Lester, & Brilleslyper, 2008; Lesh, Hoover, Hole, Kelly, & Post, 2000). Students work in teams of 2-3 and create a "toolkit" for a robot dance team captain with a team of different-sized robots. The captain needs a synchronization solution for any dance routine. This helps focus the teams on designing a general, adaptable, and explainable solution.

Teams are provided with an example dance routine and robots (Figure 1) that are carefully chosen to make visible key proportional relationships between the robots' physical design, the program parameters, and the magnitude of the robots' movements. Hence, proportional reasoning (Lamon, 2007) is a key mathematical model teams are facilitated in using. Teams are presented with the entire problem up front, but the activities are structured into sub-problems. We hypothesized that this unit would encourage mathematical modeling to a greater extent than competition activities, and that this in turn would lead to increases in students' understanding of how the robots work.

Study 1

An exploratory design experiment (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) using the RSD unit was implemented to encourage students' to use mathematics in their work with robots and to observe whether doing so led to improvements in their understanding. Identifying cases of more and less successful learning provided the basis for hypothesizing about what were key elements of the instructional design and more productive ways of connecting the mathematics to the robots.

Methods

Students from an urban middle school self-selected into one of two interventions during their elective period: preparation for a robot competition (the *Competition* group) or an MEA (the

Instruction group). Preparing for the competition requires more time, so the *Competition* group participated in 32 hours of robot activities, whereas the *Instruction* group participated in 8 hours.

The first author taught the *Instruction* sections and the school's engineering instructor taught the *Competition* section. Both groups worked with the same LEGO robots; moreover, a large part of the competition required getting their robot to move specific distances and angles, which was the primary focus of the *Instruction* group activities.

To measure understanding of the way the robot works, an 18-item, paper-and-pencil assessment of robot problem solving was designed focusing on problems involving the control of robot movements, in which proportional relationships can be applied for more effective problem solving. The items were modified from validated proportional reasoning assessments (e.g., Jansen & van der Maas, 2002; Misailidou & Williams, 2003) and given a robotics cover story (Figure 2). Both groups were given identical pre- and post-assessments at the beginning and end of their activities. In addition, the *Instruction* group activities were video recorded and the work they produced was collected, including worksheets and posters describing their evolving RSD toolkits.

Results

Learning Gains

Inspecting the assessment data, the students in the *Instruction* group made significant learning gains, $t(20) = 3.94$, $p < 0.001$, Cohen's $d = 0.55$, whereas the students in the *Competition* group did not, $t(7) = 0.58$, $p = 0.58$, Cohen's $d = 0.12$ (Table 1 and Figure 3). This supported the hypothesis that instruction encouraging the use of math in engineering strategies helps improve understanding of the system itself.

Solutions Generated

Despite overall gains, examining lower and higher quality solutions of teams within the *Instruction* group revealed meaningful differences in mathematics use. The following is a contrasting case of two teams—A2 and B1. Both solutions could be considered typical, but were chosen as representative of the solution range to present a useful contrast. Table 2 summarizes the differences, but we highlight key differences here.

The teams approached the RSD task very differently. In the first cycle, both teams generate working strategies based on relative scaling (Figure 4). However, B1's strategy is focused on a physical aspect of the robot responsible for the distance traveled (wheel size), incorporating a layer of abstraction not present in A2's strategy. In a worksheet, B1 then articulates how working strategies necessarily incorporate a multiplicative comparison between robots (Lamon, 1993), but they express an additional criterion valuing solutions that are explicit about key physical aspects of the robots.

When generating revised strategies, both teams engaged in unitizing (Lamon, 1993), based on how far the robot moves per motor rotation (Figure 5). However, B1 recognized this unit rate as the wheel circumference, but A2 did not make this connection explicit. Again, A2 was content connecting motor rotations and distance directly without incorporating physical features of the robot. In their final toolkit (Figure 6), A2 reverted back to a guess-and-check strategy, whereas Team B1 attempted to extend their scaling idea to turn movements.

Discussion

Because Team B1 included a student with prior robot experience and an advanced mathematics background, it may be that access to advanced cognitive resources (e.g., ratios and proportions) explains some of the differences between the teams. On the other hand, Team A2 was able to use relative thinking and unitizing in their intermediate solutions, suggesting that their understanding of the relevant mathematics was not a limiting factor. Instead, it seems likely that despite of having access to those mathematical resources, they felt other numerical strategies, including guess-and-check, were more appropriate.

An alternative explanation is that a key distinction between the teams was less about cognitive resources and more about epistemological resources—views about the nature of knowledge that is appropriate for particular tasks (Louca, Elby, Hammer, & Kagey, 2004). A2 was concerned primarily with getting the particular robots to be synchronized in a precise way as evidenced by their continual use of guess-and-check, and by their fine-tuning of values even after applying math-based strategies (Figure 5a). A2's view is consistent with a *calculational* orientation (Thompson, Philipp, Thompson, & Boyd, 1994). Students with a calculational orientation have a tendency to focus almost exclusively on the language of numbers and numerical operations without connecting how an understanding of the situation gives rise to those calculations. In contrast, B1 held a view of the task as being about trying to represent their ideas of how the robot works. This view is reflected in their use of wheel size in their explanations, and their defense of this approach over using distance alone. We label B1 a *mechanistic* group (Russ, Coffey, Hammer, & Hutchison, 2008). Students with a mechanistic orientation focus on identifying causal mechanisms that underlie natural phenomena. We designed a follow-up study to test these epistemological distinctions further.

Study 2

A second design experiment was conducted to test the hypothesis that epistemological orientation impacts learning. Two instructional sections were manipulated to encourage one group to take on a mechanistic orientation and the other a calculational orientation. Again, we take a resources view that epistemologies are not individual and stable, but instead are context-sensitive and malleable (Louca et al., 2004), and so can be activated using appropriate instructional moves. We expect students who adopt a *mechanistic* orientation will make greater gains in understanding relative to students who adopt a *calculational* orientation.

Methods

Students from an independent school who had just completed fifth, sixth, or seventh grade were recruited for the *Instruction* group. The *Instruction* group had two sections, each of which met five consecutive days, two and a half hours per day at a university research building. The sections were assigned randomly to conditions—one to the *Mechanistic* group and the other to the *Calculational* group. Students chose their section based on convenience. However, they were not informed of the differences between sections. The first author was the instructor for both *Instruction* sections. The typical RSD unit was implemented with both sections, except for three distinctions intended to activate the contrasting epistemological orientations (summarized in Table 3).

Teams registered for a local robot competition were recruited to serve as the *Competition* group. Two middle school teams volunteered, both of which met during an elective period. They spent a similar time allocation as in Study 1.

Based on Study 1 analyses, it was possible to use a shorter test while maintaining reliability. Twelve items with high internal consistency and diagnostic value were selected for a revised assessment. Students in all groups were given identical pre- and post-assessments. The activities of the *Instruction* groups were video recorded and their work was collected.

Results

Inspecting the assessment data, students in the *Mechanistic* group made significant learning gains [$t(9) = 3.34$, $p < 0.01$, Cohen's $d = 0.88$], whereas the students in the *Calculational* [$t(7) = 0.1.67$, $p = 0.14$, Cohen's $d = 0.51$] and *Competition* [$t(18) = 1.45$, $p = 0.16$, Cohen's $d = 0.21$] groups did not (Table 1 and Figure 7). This supported the hypothesis that a mechanistic orientation is particularly powerful in instruction focused on encouraging students to use math for understanding. It is notable that both *Instruction* groups had somewhat large effect sizes.

General Discussion

The implementations of the RSD unit in both studies helped students improve their understanding of the way the robots work. These improvements were the result of facilitating students' use of mathematics for thinking about the robots in more explicit terms, a practice less likely to occur when preparing for robot competitions. Further research controlling for time on task, using a larger number of students and classrooms, and using random assignment to minimize individual differences would be necessary to confirm these results.

In addition to providing instructional opportunities for students to use math in their engineering design, alternative ways to setup the activities influence how students approach the task. As other researchers have argued (Greeno, 2009) and investigated empirically (Elby, 2001; Hutchison & Hammer, 2009; May & Etkina, 2002; Redish & Hammer, 2009), epistemological framing is an important factor. The studies reported here add to that research base and provide evidence that framing may be manipulated in a classroom, leading to impacts on learning. Further, these studies provide clarification on how using mathematics for conceptual understanding may be more about using simple mathematics in complex ways rather than complex mathematics in simple ways (Iversen & Larson, 2006). This suggests it may not be necessary that students be provided with large amounts of mathematical prerequisites before being encouraged to use mathematics as a tool for understanding.

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Appendix A – Tables

Table 1: Pre- and post-assessment scores of robot problem solving.

	Study 1 (18 items)		Study 2 (12 items)		
	Instruction	Competition	Calculational (Instruction)	Mechanistic (Instruction)	Competition
N	21	8	8	10	19
Mean (SD)					
Pre	7.00 (4.02)	9.00 (4.57)	6.00 (2.00)	5.90 (3.25)	8.68 (2.71)
Post	9.24 (4.15)	9.50 (3.59)	7.25 (2.82)	8.50 (2.59)	9.26 (2.79)

Table 2: Summarized differences between the contrasting teams from Study 1.

Contrasting Teams	Team A2 – Calculational	Team B1 – Mechanistic
Team Composition	Sixth grade male no robot experience grade-level math	Sixth grade male no robot experience grade-level math
	Seventh grade male no robot experience grade-level math	Seventh grade male no robot experience grade-level math
	Ninth grade female no robot experience grade-level math (Algebra 1)	Ninth grade female some robot experience (out-of-school girls science program) advanced math (Algebra 1 in seventh grade)
Initial Ideas for Synchronizing Distance	Used a guess-and-check strategy	Used ratio of wheel sizes (circumference) to scale down motor rotations
	"Because if we picked anything littler than that we though [*sic*] Madonna would go to [*sic*] slow" [3 motor rotations]	"Bigger wheels go farther because one rotation is larger"
First Synchronizing Distance Strategy	Adopts a scale factor strategy based on ratio of distances, but doesn't incorporate any robot physical parameters, or references to the physical situation.	Formalizes their initial "Scale Wheel" strategy with wheel size as the basis
Explaining a Teacher Case *The case uses the ratio of distances w/ same motor rotations to scale motor rotations.*	Recognizes this as a more formal version of their strategy, but without explanation or critique	"This does work but I would rather use the wheel size because distance doesn't apply in turns and can be affected by outside factors."
Revised Synchronizing Distance Strategy	Develops a new strategy with the distance in one rotation as the basis, but without connecting that rate to the wheel size	Continues to use wheel size as the basis
	Does adjustment (fine-tuning) beyond the initial calculation	Is less concerned with getting the values exactly correct
Final Toolkit	Revert to a guess-and-check method without mention of physical robot parameters	Able to extend scale factor reasoning to turns, although doesn't incorporate the additional relevant physical robot parameters (robot width)

Table 3: Instructional differences between groups in Study 2.

Instructional Manipulation	Calculational	Mechanistic
Design Task Setup *How each task is introduced to the student teams.*	Focused on input-output transformations "Think of how to transform the motor rotations value into the desired robot distance. Create a strategy for your toolkit that is clear about each of those steps."	Focused on representing intuitions "Think of how motor rotations causes the robot to move forward a specific distance. Create a strategy for your toolkit that captures your ideas about how that works."
Teacher-Provided Cases *Example strategies given to students after they have invented their own strategies.*	Focused on identifying empirical patterns e.g., Scale factor strategy based on the ratio of the distances when using the same motor rotations."	Focused on identifying intermediate physical quantities e.g., Scale factor strategy based on the ratio of the wheel sizes.
Instructional Support *Questions instructors use to assess and advance students when they are inventing their own strategies.*	Focused on correctness of calculations "What are the steps you took to get this value?"	Focused on connecting quantities and operations to the physical situation "What does this value/operation correspond to on the robot?"

Appendix B – Figures

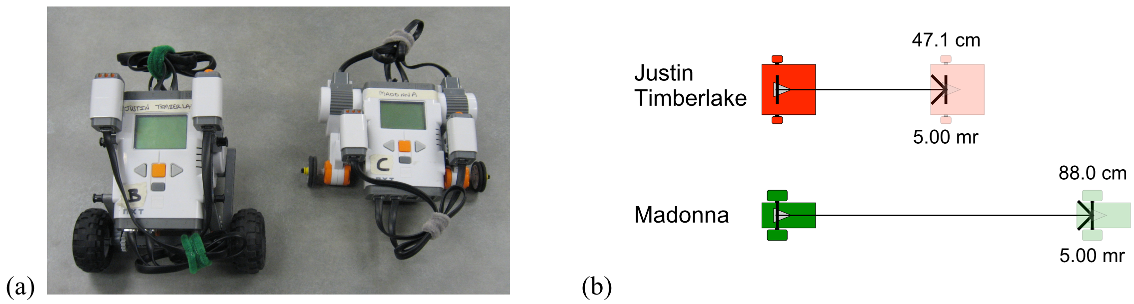


Figure 1: (a) the contrasting robot designs used in the RSD unit and (b) a representation of their lack of synchronization on the first move of the example dance routine.

A robot completes a move with 12 motor rotations and moves forward 14 centimeters. You modify the program to be 30 motor rotations. How far will it move forward now?

Answer: ____________________

How did you find your answer? Please show your work and explain in the space below.

Figure 2: An example item from the robot problem solving assessment.

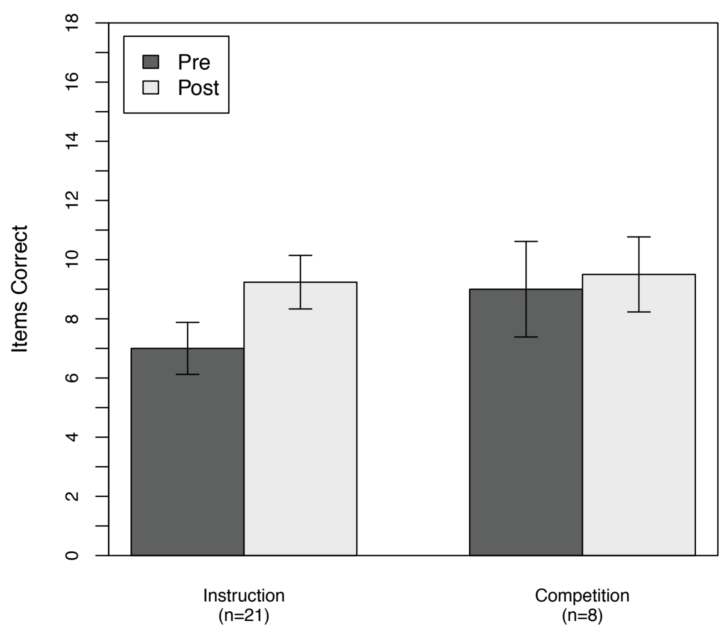


Figure 3: Mean scores (+SE) on pre- and post-assessments of robot problem solving for Study 1.

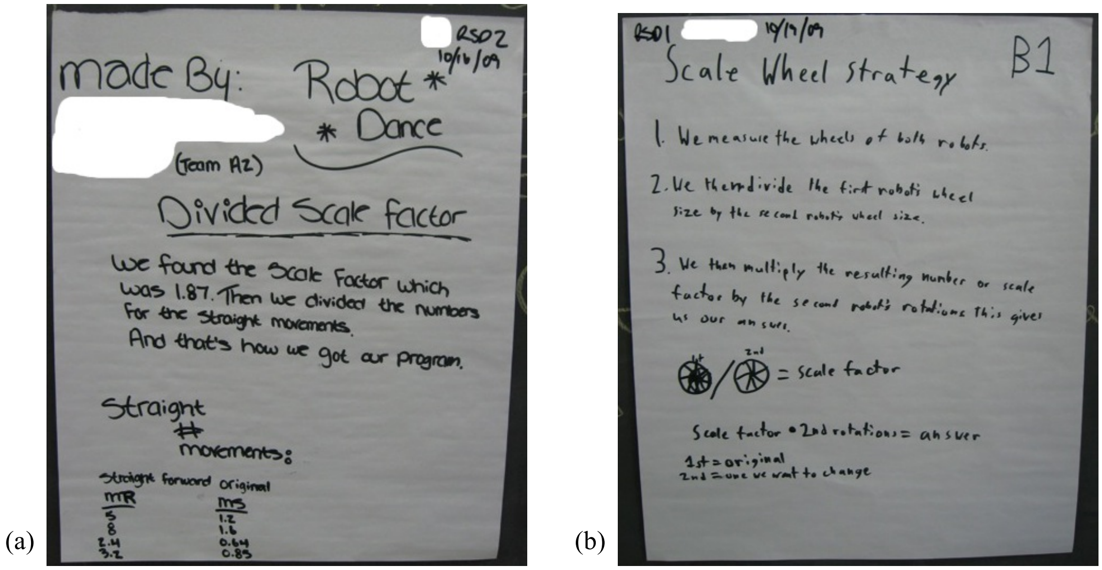


Figure 4: Posters of first strategies for synchronizing distance generated by the contrasting teams, (a) Team A2 and (b) Team B1.

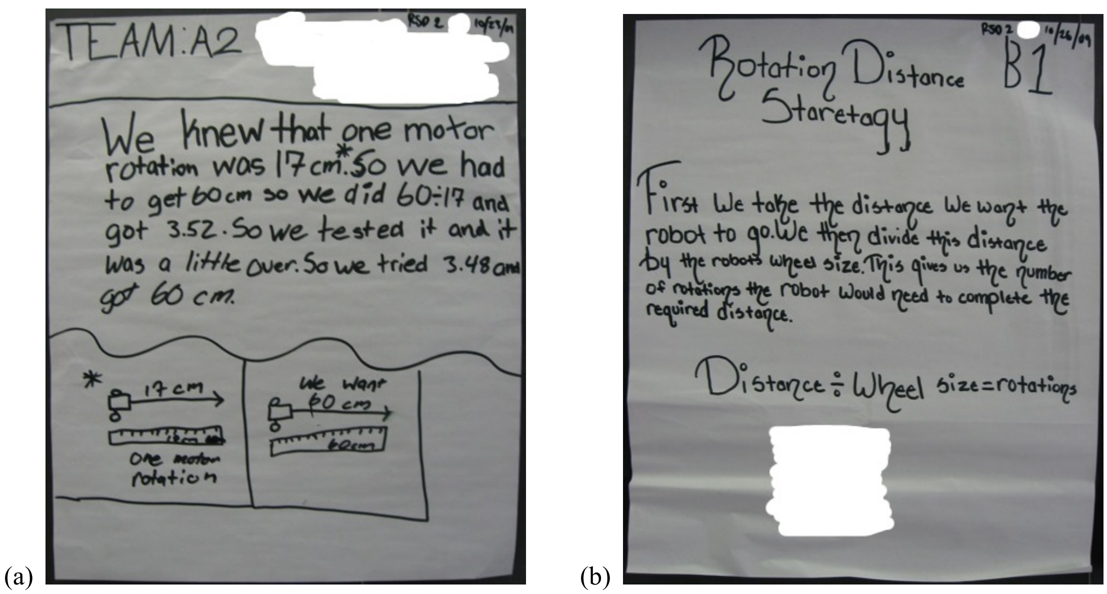
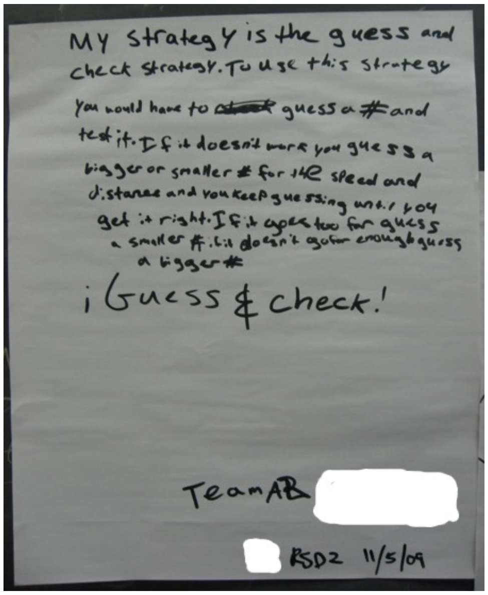


Figure 5: Posters of revised strategies for synchronizing distance generated by the contrasting teams, (a) Team A2 and (b) Team B1.

(a) 

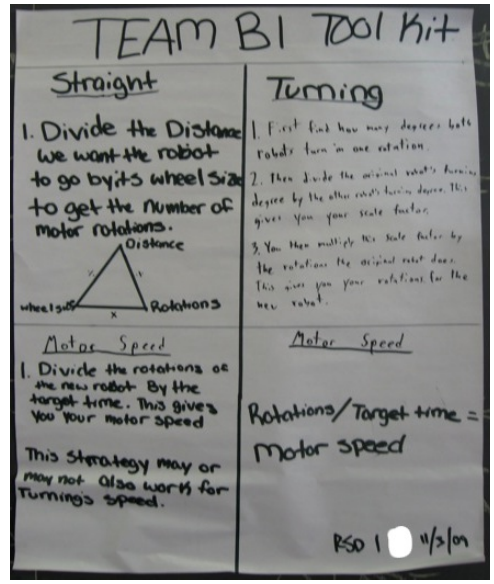
(b) 

Figure 6: Posters of final toolkits for synchronizing robots generated by the contrasting teams, (a) Team A2 and (b) Team B1.

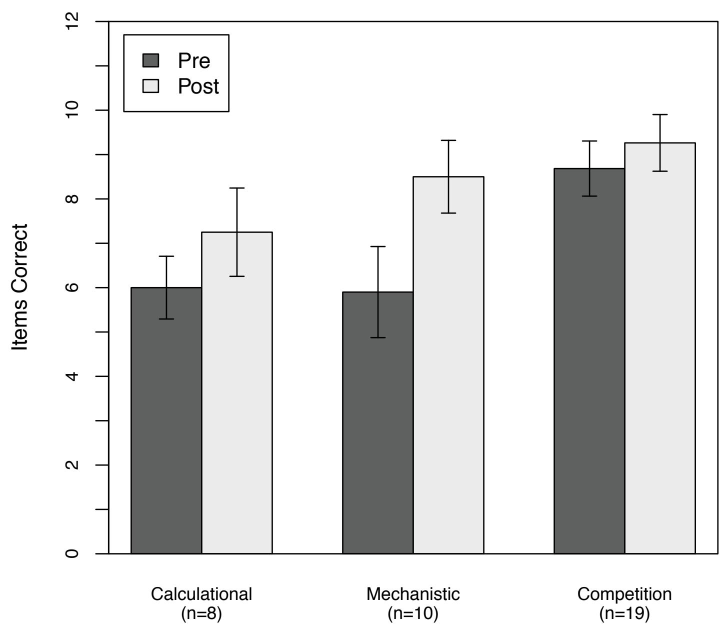


Figure 7: Mean scores (+SE) on pre- and post-assessments of robot problem solving for Study 2.