RESOURCES FOR LEARNING ROBOTS: ENVIRONMENTS AND FRAMINGS CONNECTING MATH IN ROBOTICS

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CONNECTING MATH IN ROBOTICS

Eli Michael Silk, PhD

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How do learning environments influence the ways that middle school students use math to engage with and learn about robotics? Data from two observational studies suggest that existing formal (scripted inquiry) and informal (competitions) learning environments in this domain are limited in their support for connecting math with robotics. In light of the evaluation of these existing learning environments, two additional studies were conducted documenting the design, implementation, and redesign of a new learning environment intended to more effectively align learning and engagement with the connection between math and robots. Pre-post assessments and analyses of student work support the hypothesis that a model eliciting learning environment can facilitate learning while maintaining interest in both disciplines, and facilitate the development of a greater sense of the value of math in robotics. Two additional studies expanded on the previous work. The first study identified two contrasting approaches for connecting math with robots in the context of the model-eliciting learning environment from the previous studies. One approach used mathematics as a *calculational* resource for transforming input values into desired output values. The second approach used mathematics as a *mechanistic* resource for describing intuitive ideas about the physical quantities and their relationships. The second study manipulated instructional conditions across two groups of students that encouraged the students

to take on one of these approaches or the other. Both groups engaged in high levels of productive mathematical engagement: designing, justifying, and evaluating valid strategies for controlling robot movements with connections to mathematics. But only the mechanistic group made significant learning gains and they were more likely to use their invented robot math strategies on a transfer competition task. All six studies taken together provide a rich description of the range of possibilities for connecting math with robots. Further, the results suggest that in addition to carefully crafting environments and associated tasks to align math and robots, that instructional designers ought to pay particular attention to helping students frame their approaches to using math productively as a tool for thinking about situations.

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PREFACE

I would like to express my gratitude to so many people without whom the completion of this dissertation would certainly not have been possible. Thank you to the students who worked so hard when it wasn't required of them. Thank you to their parents who sought out challenging opportunities, allowed their children to participate, and made sure they showed up. Thank you to the teachers, coaches, directors, and administrators who graciously welcomed me into their classrooms and programs. Thank you to the Robotics Academy for introducing me to the wonderful world of educational robotics and providing me a well-traveled path for navigating through it. Thank you to the Schunn Nokes Lab for regularly being willing to listen and provide me with feedback on my very rough ideas. Thank you to my committee for their willingness to engage with my ideas and push me to move them forward. Thank you to Chris Schunn for providing substantial and unwavering support at every level throughout this entire process. And thank you to my family for giving me the time, space, and encouragement to see this through.

1.0   INTRODUCTION

This research is located in the broad field of the learning sciences with a particular focus on the cognition and learning involved in coordinating between the disciplines of mathematics and robotics. The goal of this dissertation project was to investigate the alternative ways in which mathematics might be positioned within a task involving physical and technological components—in this case, controlling basic robot movements. Particular attention was paid to investigating math as a useful tool for problem solving and improved understanding within that situation. Features of designed learning environments were evaluated in terms of the extent to which they facilitated students in valuing and using math in their robot problem solving.

Robotics as a fun and challenging engineering and technological design activity

Robots are an increasingly common context for K-12 students to participate in engineering and technological design activities. The largest example of this sort of activity is the robot competitions sponsored by FIRST (For Inspiration and Recognition of Science and Technology). FIRST estimates that over 212,000 students from age 6-18 participated in their four levels of robotics programs in 2009 (FIRST, 2010). A stated goal of FIRST is to get young people to celebrate students who build and innovate with engineering and technology with the same energy and enthusiasm that is more commonly reserved in today's culture for professional athletes and entertainers. As the title of a recently-released popular-press book—*The New Cool* (Bascomb,

2011)—suggests, robotics competitions may be a promising context for encouraging students to pursue further study and careers in STEM (Science, Technology, Engineering, and Mathematics) fields precisely because the competitions effectively blend a focus on excitement and fun with an opportunity to engage in an extended and challenging activity from which to learn valuable STEM-related skills. When students have positive experiences participating in robot competitions, they then may be more likely to build sustained interest in STEM fields, to elect to take additional and more advanced courses in high school and college, to persist in those courses even when they are difficult, and then to seek STEM-related career opportunities. Indeed, when compared to a matched comparison group from an existing national dataset, alumni participants of the high-school level FIRST Robotics Competition (FRC) were more likely to attend college, to major in a STEM-related field, and to expect to pursue a STEM-related career (Melchior, Cohen, Cutter, & Leavitt, 2005). Evaluations of the FIRST program have been primarily concerned with measuring that program's impact on fostering confidence and sustained interest in robotics (Melchior, Cutter, & Deshpande, 2009). But, building STEM-related skills is also a stated part of FIRST's mission, and it is ultimately a combination of both interest and competence that is likely to impact students' STEM-related education and career choices (Wigfield & Eccles, 2000).

The potential of robotics as a context for students to participate in challenging engineering and technological design opens many questions of value to the learning sciences and STEM education research fields. One such question is whether and how students' initial interests in working with robots and in "making the robot do what I want" (Petre & Price, 2004) might lead to the development of more general understanding about the robot context and problem-solving strategies for navigating it. It is not yet certain what are the most effective ways to

structure these particular experiences in robotics—or more general experiences in other challenging engineering and technological design contexts—so that even while sustaining or enhancing students' interest in STEM, students also become more competent designers in those specific contexts and acquire more general understandings that can be flexibly and innovatively applied in other similar situations.

Coordinating engineering and technological design with mathematical thinking

In these robot competitions and in K-12 settings more generally, although some exceptions almost certainly do occur, traditional boundaries between disciplines make it unlikely that if a student has an experience doing some sort of engineering or technological design that they will be encouraged to draw on mathematics to help understand, justify, revise, or communicate their design ideas. They are more likely to focus their efforts on building a solution and tinkering with it till it satisfies some unspecified criteria (and then demonstrating it without explanation). Not drawing on mathematics as a tool for designing is most likely even more common at the middle and elementary school level, when students are first being introduced to these robot competitions and may be perceived as not having a sophisticated enough background to use math in their designing. But authentic engineering design is characterized more by systematic design accompanied by careful quantitative analyses utilizing both given and invented mathematical models of the physical reality and the technological adaptations available for working within that reality (Gainsburg, 2006). Understanding the ways in which a novice designer is able to coordinate aspects of specific situations—a physical sense of the situation and awareness of the technological tools for adapting it—with models of the general structural characteristics of those situations—a mathematical sense of the situation—is therefore important to inform the design of

instructional activities that better prepare students for participating in authentic engineering and technology practice.

Cognitive and epistemological resources perspective

One theoretical perspective for examining this issue is a cognitive perspective in which the conceptual resources that students draw on in design tasks determine the learning outcomes that result. Cognitive resources may include knowledge components about important features of the situation, the relationships between features, and strategies for solving problems that act on these features (Siegler & Chen, 2008). One interesting finding is that informal strategies that are attuned to specific aspects of problem-solving situations are valid and in many cases less error prone than more efficient, abstracted, formal procedures (Koedinger, Alibali, & Nathan, 2008). A prominent example is a so-called guess-and-test strategy, or empirical solution method, which contrasts with more formal analytic methods that usually involve algebra (Levin, 2009). This guess-and-test method, similar to other informal strategies such as building-up methods in proportional reasoning (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998), is successful because students are acting on quantities with well-understood referents in the physical situation and are therefore unlikely to make abstract errors that would violate situational constraints (Nhouyvanisvong, 1999). As a result, although these informal strategies may be limited in their applicability as the complexity of the task increases, their meaningfulness and usefulness within certain limited situations can serve as a cognitive resource for building more sophisticated and powerful strategies. The cognitive perspective suggests that a careful task analysis of students' strategies for problem-solving in robotics and how those strategies can be generalized to more sophisticated methods could help guide the design of effective learning activities.

An alternative perspective, emerging from situative accounts of learning, recognizes that students have many different cognitive resources that have the potential to be productively used in different situations, but which cognitive resources students activate is very much dependent upon their perception about what sort of understanding is called for in the task (Hammer, Elby, Scherr, & Redish, 2005). Greeno (2009) has referred to this as epistemological framing, and calls for more explicit attention to resources available to the student in this sense. For example, two ways in which students may frame their activity are a conceptual orientation versus a calculational orientation (Thompson, Philipp, Thompson, & Boyd, 1994). In a conceptual orientation, an individual is focused on making meaning and connections with more general structural aspects of situations that may apply beyond the specific instance. In a calculational orientation, the ultimate objective is to obtain a particular answer to a particular problem, and so efforts are focused on that narrower goal. diSessa (1985) provides a rich description of these contrasting epistemologies by analyzing two MIT freshman taking a freshman physics class. The first case—"Results Man"— was focused on numerical results and solutions to problems without recognizing the value of qualitative analyses or connecting to intuitive understandings. The other student—"Real Understanding"—employed a problem-solving process that put much more emphasis on making sense of what was going on in the situation before applying any equations or focusing on numerical results. This student expressed that his ultimate goal for problem solving was to figure something out about the world and not "getting a number" (diSessa, 1985, p. 104). Thus, according to an epistemological framing perspective, activating relevant cognitive resources for making sense of robot design tasks may be ineffectual if students view the task as requiring them only to design a particular solution to a particular problem rather than to develop a more general understanding.

The cognitive and epistemological perspectives together may provide a sufficient framework for investigating the opportunities and challenges that middle school students may have as novice designers in a robot context. In this context, students may be encouraged to coordinate their intuitive understanding of the physical reality and of their technological tools for adapting it with mathematical models that help them to understand, justify, and revise their designs.

Why robotics?

In addition to the increasing popularity of robot competitions, robotics was chosen as the context for this study for a number of reasons that make it a discipline especially suited for investigating the role of mathematics in the development of physical and technological understanding for improving design solutions. In this study, I focus further within the discipline of robotics on how students come to understand and design simple robot movements. The first reason this focus on robot movements is appropriate is the high occurrence of non-mathematical guess-and-test strategies employed by novice students when attempting to program the robots to move straight specified distances or turn specified angles in specified amounts of time. At the same time, this context is appropriately modeled using concepts of proportional reasoning—a foundational idea in middle school mathematics (Lamon, 2007)—relating the quantities of the physical construction of the robot to quantities used to program the robot in predicting the magnitude of the robot movements. By using proportional reasoning strategies, students can more efficiently program their robots to move in particular ways that they specify, but can also more flexibly adapt their strategies to different movements and robots of different physical designs. A related advantage of this context is that proportional reasoning consists of a wide range of informal and

formal strategies (and valid and invalid strategies) that are well studied in the literature both in formal schooling (Ben-Chaim et al., 1998) and in real-world contexts (Hoyles, Noss, & Pozzi, 2001). Thus, the context affords a wide range of strategies, both non-mathematical and mathematical that vary in their power for guiding design solutions.

Another advantage of this robot context for investigating students' coordination of mathematical thinking in their engineering and technological design is the blend of complexity with control in a real-world context (Schauble, 1996). The robots are reliable, manipulable, and inspectable. At the same time they are not simply a made-up or imagined entity, but instead they do real things out in the physical world, and so a person's own intuitive knowledge of the physical world will apply to the robots as well.

The plan for the dissertation

I pursued the goal of investigating how beginning robotics students coordinate mathematical thinking in their robot designs in this dissertation project in two parts. Each part had a different research focus and method, but both made use of a mix of quantitative and qualitative analyses. In the first part, I used both observational and design experiment methods to identify the nature and extent of the ways that students connect math in robotics and to analyze the effect of designed environments for supporting students' learning of those connections. The focus of the investigations was on identifying features of environments that were not only effective at helping students learn those ideas, but also at maintaining their interest in both disciplines and enhancing their sense of the connections between the disciplines. The findings from this part suggest that activities which are carefully designed to favor strategies that include math as a central rather

than supplemental part of the activity have the best chance for achieving learning gains while sustaining engagement.

In the second part, I used an experimental research design to pursue a more differentiated understanding of the ways students connect math with robotics, and how they may connect math with physical situations more generally. This included identifying alternative ways students made this math-to-situation connection, manipulating the environment so that students took up these different ways, and investigating the results when they did so. The findings from this part suggest that the most common way for students to connect math with robotics is as a way to work with and manipulate the numbers in the situation. Although less common, the alternative way consists of students who frame the use of math as a representational tool for being explicit about their ideas of the way the robots work. This alternative way for connecting math in robotics leads to greater learning and a deeper sense of the connections between the two disciplines.

Taken together, this dissertation project provides an initial map of the landscape of the place of math in introductory robotics. The results have implications for cognitive and learning science research and for research in STEM disciplines that are focused on understanding how students integrate knowledge across disciplinary boundaries. In addition, the findings may be useful and informative for designers of learning environments for introductory robotics and the teachers and coaches responsible for guiding their students through those environments.

2.0   PART 1 – ENVIRONMENTS FOR LEARNING ROBOTS

The major issue addressed in this first part of the dissertation project is how to motivate systematic analysis of situations in a learning environment where students may initially approach the situation in a much less formal manner. In most environments for learning robots, students are free to choose to participate or not, because these environments are rarely a part of the standard school curricula. Robots, although often thought of as having educational potential (Petre & Price, 2004), are more likely to be used in elective activity periods during school or in after-school enrichment programs than as a part of core content in formal school classrooms. In these settings, students choose to engage in an activity involving robots because they have some interest in technology in general or in robotics in particular. But the students' initial interest is often at a level associated with fun rather than work, and so they may be resistant to efforts that push them to engage at a more reflective and conceptually difficult level. It is thus a challenge to designers of these environments to maintain that interest while also bridging that interest into a deeper, more conceptually productive form of engagement. The design of learning environments that lead effectively to productive disciplinary engagement (Engle & Conant, 2002) is challenging and not well specified for well-studied formal classroom environments that target core disciplinary areas, and so designing environments that target the learning of robotics is no exception.

The first part of this dissertation was set within the context of a reporting of the design history of the *Robot Synchronized Dancing* (RSD) unit. The goal of the RSD unit was to be a learning environment for introductory robotics that effectively helped students connect math with robots while sustaining their engagement in both disciplines. In the development of this particular unit, along with investigations of contrasting units, there were lessons learned about the broader set of opportunities for learning introductory robotics. These lessons learned have implications for more general issues related to the design of environments that push students to be more intentional learners while maintaining high levels of engagement.

This part of the dissertation included four studies, each focused on identifying the features of an environment that influence the connection of math in learning introductory robotics. Study 1 and Study 3 were observational studies in the context of already-established environments that target introductory robotics. These two contrasting environments—a formal classroom unit and an informal competition setting—helped to define the space of problems that students need to solve in this discipline and their common solution approaches. The contrasting environments also helped to define instructional possibilities for facilitating those solutions. Study 2 and Study 4 were design experiment studies in the context of a first and then a revised version of the RSD unit. Embedded in this design and redesign were conjectures about the key features of environments that promote learning while maintaining engagement.

2.1   OVERVIEW OF STUDIES

Because of the breadth of territory covered in this part of the dissertation project, the following is a brief summary of the main findings from each study and the connections between them, which can serve as a guide for the reader:

Study 1 – Scripted Inquiry.   I examined a formal classroom unit, in which the explicit instructional goal was to help students learn math concepts. Observing in this environment, I found that although the activities were structured such that students did attend to math, in many cases the connections to math were misaligned, decontextualized, and primarily procedural in nature. There was little development of the usefulness of the math for actually doing things with robots even though the robots were the sole context within which the math was being targeted. As a result, the students did make some learning gains, but also ended with a diminished level of engagement in robotics and math and a limited view of the connections between them.

Study 2 – Design Based (RSDv1).   I designed an alternative formal learning environment, in which the goal was to more explicitly highlight the value of math as a tool for solving robotics problems. Designing this environment provided an initial test of the idea that better aligning the math ideas with actual robot design problems would not only lead to learning, but also to maintained engagement in robots and math, and a stronger sense of the connections between them. Although the design experiment did not include measures reliable enough to test this idea quantitatively, qualitative assessment of students' participation in the unit suggested that they indeed did engage with challenging math in the service of solving their robot problems. Furthermore, careful inspection of their ideas helped to better understand the particular ways in which the more general math of proportional reasoning is situated within both a developing and

more sophisticated understanding of controlling robot movements. This led to possibilities for redesigning the unit to more immediately and more substantively build from students' common ideas and strategies that do include math and that are productive building blocks for more sophisticated understanding.

Study 3 – Competition. With a better understanding of the concepts and strategies that students employed to connect math with controlling robot movements, I observed an informal environment for learning robots—a robot competition—to investigate the extent to which those same ideas are present and useful in that context. Indeed, the problem of precisely controlling robot movements was a central aspect in the competition tasks, but in contrast to the formal environments, student teams rarely attended to the relevant math when inventing their solutions. The design of the competition environment favored fine-tuned solutions regardless of consideration of the more general ideas. Unsurprisingly, students were highly engaged in creating solutions and maintained their interest in robots as a result of preparing and participating in the competition, but they did not exhibit gains in learning. On the other hand, there were cases of teams that did appear to use math, some successfully and others unsuccessfully, but when they did make an attempt, it did result in learning in addition to maintaining engagement. This suggested that when students chose to use math in context, it was possible to obtain positive effects on both learning and engagement simultaneously.

Study 4 – Model Eliciting (RSDv2). The observations in the competition setting suggested that the challenge for environments focused on learning robots was not just general conceptual math difficulties. They were also about helping students move beyond a tendency to focus on developing fine-tuned solutions for particular problems in place of attempting to understand the more general features and underlying structure of the way the robots work. In this

study, the RSD unit was redesigned using a model-eliciting activity framework so that the focus of students' activities in the unit would be more clearly and more immediately aligned with the goal of attending to the general structures of the problem. This redesigned unit resulted in similar learning gains as the scripted inquiry unit but also resulted in a positive change in the perception of the value of math for robotics. The model-eliciting framework thus served as a closer approximation to the ideal of "hard fun"—an environment within which the engaging part for the student is the learning itself.

2.2   BACKGROUND

2.2.1   Learning and engagement

Engle and Conant (2002) describe the goal of instruction as productive disciplinary engagement, possibly implying that engagement is necessary for learning. Alternatively it may be that engagement is better characterized as a dimension independent of learning. In this model, learning and engagement may be thought of as orthogonal dimensions that define a space of outcomes for learning environments. Figure 1 illustrates this two-dimensional space as well as characterizations of the sorts of activities that would be likely lead to outcomes within that space.

A main challenge in this space from an instructional design perspective is to figure out how it is possible to balance pushing students to both use more systematic, explicit, and generalizable strategies in their problem solving while maintaining their engagement – what can be called "hard fun".
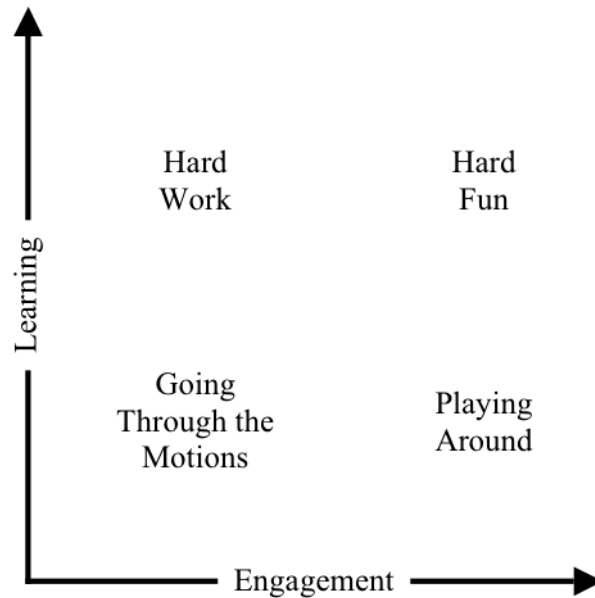
Figure 1. Theoretical space of learning and engagement with typically associated activities

2.2.2   Opportunities for robotics learning environments

Robots provide an interesting case of a discipline that integrates and connects with many other disciplines. In this dissertation project, I focus on coordinating aspects of math as a tool within the robot domain that actually helps understanding and designing with the robots. Math may be used as a tool for understanding a physical situation in a number of ways. For example, numerical analysis of empirical data may help students to separate explainable patterns from random error, and algebraic modeling of structural features and their relations may help students to be explicit about what aspects are relevant and the implications when those aspects are modified.

Schwartz et al. have provided compelling examples of how math may be used as a tool for thinking about developing knowledge of physical situations (Schwartz, Martin, & Pfaffman, 2005; Schwartz & Moore, 1998). By adapting situations minimally, such as by making quantities harder or easier to measure and simply prompting to "show your math," Schwartz et al. have shown how the math aids in developing understanding.

But questions remain about whether situations themselves have these properties of being amenable to improved understanding through the application of math. In which case simply setting up the conditions where students would be encouraged and motivated to explicitly use math in a robot context should be sufficient to facilitate developing understanding. An alternative view, however, is that the ability of robots as a context to realize the benefit of math as tool for thinking about the situation is entirely dependent upon the features of the environment in which the math is being used. I will set out to show that the answer is more likely and to a greater extent the latter one. Furthermore, the key features of learning environments that make it possible for robots to realize these opportunities to connect with math are subtle to get right from an instructional design perspective.

2.2.3   Challenges for robotics learning environments

Four challenges and associated questions guide the evaluations of environments for learning robots in this dissertation project:

1. **Focused content.** How do you make it so the activity that students actually do aligns with the disciplinary ideas that are the intended targets?

2. **Motivated activity.** How do you get students to actually care about the task and want to see the result?

3. **Accessible problems.** How do you make the task accessible so that students actually can "see" the problem that needs to be resolved?

4. **Useful resources.** How do you provide them with the resources (information and tools) they need to solve it?

2.2.4 Introductory robotics as controlling robot movements

In this section I identify more clearly the particular set of robotics challenges that are the focus of this dissertation project, as well as the particular math ideas that can be used as tools to think about and solve these robotics challenges. I used the LEGO® MINDSTORMS® NXT 2.0 robot platform as the context for this research as it is the most popular platform for students to get introduced to mobile robots. The robot itself comes as part of a kit that includes building parts, wheels, motors, sensors, a battery, and a microcomputer brick. There is also associated drag-and-drop software for programming the brick. Although the robot parts can be configured in countless ways, a typical configuration and the one used here is as a robot with two wheels each connected to a separate motor so that they can be controlled independently (Figure 2). The wheels are connected directly to the motors so that each rotation of the motor corresponds to one rotation of the wheel. A third, smaller wheel is set up in the back of the robot for balance. It is set up on a pivot so that it will automatically align itself in the direction of motion and does not need to be powered by a motor. To move straight forward, both motors are programmed to move in the same direction at the same speed. To turn, the motors are programmed at different speeds.

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For example, if the right motor is turned on, but the left motor is still, then the robot will pivot around the left wheel and make a turn to the left. There are a number of sensors that can be attached, such as a touch sensor, a sound sensor, and a light sensor, and these sensors can be used to make actions by the robot conditional upon some sensed event. For example, a robot could move forward until its touch sensor is triggered, such as when it runs into a wall.

Figure 2. LEGO MINDSTORMS NXT 2.0 robot

Many aspects of learning about these robots could have been the focus. Introductory robotics includes a wide range of varying challenges that roughly correspond to the categories of either building or programming challenges. In building, the goal is to design structures (both the robot's base and its attachments) that function stably, efficiently, and reliably under weight stress (the robot itself plus the objects in the world it manipulates) and when the robot and its parts are in motion. The programming aspect is focused more on specifying actions that break down and solve multi-step goals in environments that are dynamically changing while the robot's position within that environment is also changing. Each of these categories may be broken down further in many ways and both are important for success in introductory robotics. Indeed, research that

focuses on relating proportional reasoning to robots through issues of building, such as gear ratios, has been conducted (Norton, 2006). However, a smaller and narrower aspect of introductory robotics was chosen as the focus of this dissertation project based on the criteria of identifying a challenge that is both a common one that students encounter and an accessible one in terms of being open to lots of solution approaches from students with and without prior robotics experience.

All students who work with mobile robots must attend at some level to solving the problem of how to program the robot to move forward a certain distance and to turn in a certain direction. Also, although less common, they have to figure out how to do each of those moves in a certain amount of time. This aspect of controlling robot movements is the most basic challenge in introductory robotics, but nevertheless one that is not trivial. Even further for the purposes of this research, this challenge affords a range of solution approaches that are both mathematical and non-mathematical in nature, so it is an opportunity to investigate what influences the types of strategies that students use. For this dissertation project, I approach this task from the programming perspective in the sense that I assume the building of the robot has already been completed, and so the challenge is to figure out how to program the robot so its movement actions are the intended ones. I also minimize the relevance of the programming logic, in the sense that the sequence of conditions and actions is not the primary issue. Ultimately, the problem is reduced to figuring out how many motor rotations and what motor power level is needed for each desired movement.

Students typically program these robots using an associated drag-and-drop interface. Figure 3 is a screenshot of this interface. Students can drag blocks to the middle grid-like portion of the interface called the Programming Area. This indicates what action they want the robot to

perform and in what order. In this case, one move block has been dragged to the Programming Area. When the student clicks on a programming block, the parameters for that block appear at the bottom of the interface in the Configuration Panel. For the Move Block, students can control the direction of the motors, whether one or both motors should be powered, the power level, and the duration. The duration indicates how long to rotate the motors and can be set in units of rotations, degrees, seconds, or unlimited (to make the motor duration conditional upon the result of some other block). When the program is complete, the student connects the robot to a computer with a USB cable and downloads the program to the robot by pressing the download button located in the bottom right of the Programming Area. They then get the robot ready to run, and navigate through a menu system on the robot brick to start the program.
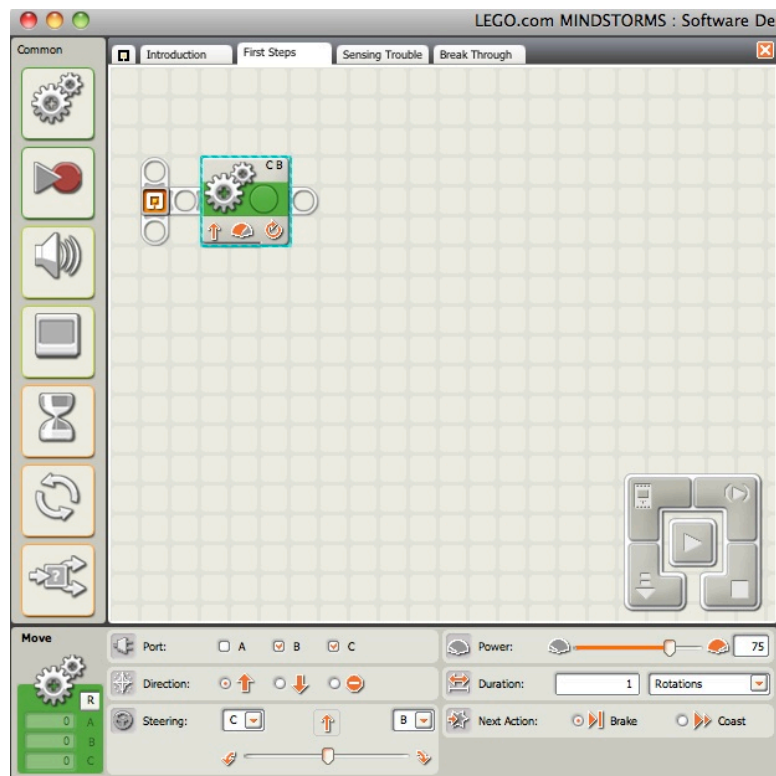
Figure 3. LEGO MINDSTORMS NXT programming software

Although programs can get very complex when they include iteration and conditionals, in the cases considered here students programs consist mostly or entirely of a simple sequence of move blocks. Within each block, students have to make an intentional choice about what duration value and power level to use so that their robot moves in the desired way.

2.2.5 Robot movements and the math of proportional reasoning

In deciding the number of motor rotations and the motor power level to make their robots move certain amounts, students can approach the task using a variety of strategies, some of which may take advantage of the underlying structure of the situation more than others. Figure 4 is an illustration of this task to help contextualize the strategies that are possible and their connections to math. However, before I review particular strategies, I review a number of math concepts that are relevant to the task, all of which can be seen as component concepts of the big idea in mathematics of proportional reasoning (Lamon, 2007).

40 cm

8 motor rotations

(a)

60 cm

? motor rotations

(b)
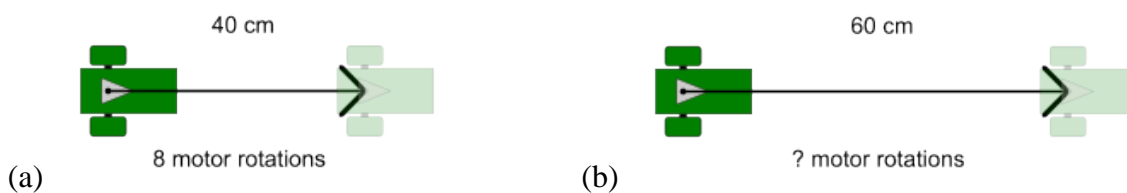
Figure 4. The basic robot movement problem

The ability to reason proportionally is a culmination of elementary school math focused on arithmetic. At the same time, it is a critical building block for high school level math and science beginning with algebra and extending far beyond (Lesh, Post, & Behr, 1988). As a result,

problems that involve proportional reasoning are especially suited for middle school age students, but can be accessible to students in upper elementary school and can also be challenging for high school students and adults. Proportional reasoning is conceptually demanding because it requires one to think carefully about what is changing and what is staying the same from one situation to the next, to describe relationships between quantities in multiplicative terms rather than in additive terms, and to keep track of multiple pieces of information at one time. In addition to being a foundational mathematics concept, proportional reasoning relates to a wide range of situations in everyday life (Schliemann & Carraher, 1992) and in the workplace (Hoyles et al., 2001), such as those that involve unit rates, mixtures, or scaling (Langrall & Swafford, 2000). Proportional reasoning is also central in understanding how robot movements can be controlled, as the relationships between the physical construction of the robot, the values used to program the robot, and how the robot actually moves are often proportional in nature.

The first math concept that relates to this situation is the very general idea of *covariance and quantitative relations*. This math concept involves a recognition that as one quantity changes there is a corresponding change in another related quantity. This sense of two quantities varying together is not necessarily causal or directional. The fundamental aspect is being able to identify and differentiate two quantities, each with their own distinct measure, and then be capable of observing, predicting, and manipulating one with the expectation of change in the other. A student might represent that covarying relationship verbally. For example, in the situation in Figure 4, a student might respond that as the motor rotations increases the distance the robot moves forward also increases. Other sorts of representations might include a mathematical equation in which a variable corresponding to one quantity is on one side of the equation and a

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variable corresponding to the other quantity is on the other side of the equation. Alternatively, a student could capture the covarying relationship in a table that shows how the two quantities change together in a variety of instances. Difficulties that students have with covarying relationships may be less about the general idea that quantities can change together, and more about being able to isolate the quantity from other related aspects. For example, when trying to decide which of two characters were traveling faster, high school students suggested that both characters went the same speed because they both went the same number of steps in the same amount of time (Lobato & Thanheiser, 2002). This response suggested that they were focused on speed in terms of how fast legs move, rather than the intended aspect of the situation, which was how fast the whole objects (the characters) were moving in space. Similarly in this context, students need to be able to distinguish a robot's movement in space from the movement of its wheels. This may be especially confusing when working with turning movements, since in that situation both the wheels and the robot's body are turning and the student needs to differentiate those two to figure out how they relate to each other.

The second math concept that relates to this robot movement situation is the idea of *relative change*. This math concept involves a sense that in some situations it is less appropriate to ask "how *many*" more (or less) of some quantity there is in one situation compared to the next, since that suggests that the relevant contrast between quantities in a situation is some absolute count (Lamon, 1995; Sowder et al., 1998). Again referring to the situation in Figure 4, our prior experience with this robot is that it moved forward 40 centimeters, so a student might respond that there are *20 more* centimeters to move forward in this move compared to the previous move. Instead, it is more appropriate to approach some situations by asking the question "how *much*" more (or less) of some quantity there is in one situation compared to the next (Lamon, 1995;

Sowder et al., 1998). This suggests that the relevant contrast between quantities in a situation is some relative amount. Back to the example situation, a student who has recognized that relative thinking would better apply in this robot situation might instead respond that the robot has to move forward *1.5 times* the number of centimeters in this move compared to the previous move. A student with a strong understanding of relative change would be able to employ multiplicative operations (multiplication and division) rather than additive operations (addition and subtraction) in their representations of the relationships between quantities and in their problem solving when using those quantities. Correspondingly, a student would reason about the situation with mental operations of iterating and scaling (multiplication operations) and partitioning (division operation). However, even though a student may be able to recognize and reason with relative change in simple scenarios, such as when they want a robot to move twice as far as it did a previous time, they may have difficulty applying those same ideas to situations that are more complex numerically. Research on levels of proportional reasoning and associated strategies suggests that indeed problems that involve integer ratios are easier for students than those that involve non-integer ratios (e.g., ratios of 3/2 or 5/2), and even that problems involving halving and doubling are easier still (Misailidou & Williams, 2003; Tourniaire & Pulos, 1985). Hybrid sorts of reasoning are possible as well. For example, a strategy observed in some situations is that a student will attempt to use a multiplicative strategy in a non-integer ratio problem by scaling up from one situation to the other using the nearest integer multiple, but then will fall back to an addition strategy to handle the remainder (Misailidou & Williams, 2003; Tourniaire & Pulos, 1985). In the robot context, very few of the measurable quantities turn out to be "clean" numbers, and so to apply relative thinking effectively, students will have to be able to problem solve with and reason about non-integer relative amounts.

The third math concept that relates to this robot movement situation is the idea of *invariance*. Although related to the concept of covariance, invariance suggests a different idea. This concept is about recognizing that even as some aspects of the situation are varying, other aspects of the situation stay the same. The invariant aspects of a set of situations can be used to apply knowledge about one instance of the situation (or a set of instances) to a new instance in which some aspects are unknown. In proportional situations, the invariant relationship between quantities is always of a multiplicative nature, either a ratio or a product of two quantities. In the situation in Figure 4, a student might recognize not only that the new distance is some relative amount more than the previous distance (1.5 times as far), but also that that relative amount should be the same for the corresponding quantity of motor rotations. Hence, the number of motor rotations for the robot to move forward the new distance should also be 1.5 times as many motor rotations as in the previous move (12 motor rotations). Other invariants exist as well. In fact, in directly proportional situations there are two distinct invariant aspects. The first one is that the relationship comparing two different instances of the same measure, referred to as a comparison *within* measure spaces or a *scalar* operator, is invariant across the two instances. This is the 1.5 times aspect of the example. A second aspect is that the relationship, referred to as a comparison *between* measure spaces or a *functional* relationship, is also invariant. In the example, because the same robot is being used in both moves, and because it moves at a constant rate, the ratio of motor rotations to distance should also be the same in both moves. Simplifying the ratio in this case it would be reasonable to conclude that since for 8 motor rotations the robot moves 40 centimeters then for every 1 motor rotation it must have moved 5 centimeters. Applying this same functional relationship to the new move, it is possible to figure out that using 12 motor rotations to move forward 60 centimeters would preserve that same 1 to 5 relationship.

A student with a strong understanding of invariance in proportional situations would be able to recognize and use both invariants within and between measure spaces. In the robot context, the functional rate also has an additional correspondence in the situation, since how far the robot moves in one rotation is equal to the distance around the wheel (its circumference). A number of researchers have explored how students (and teachers) come to understand and use a *ratio-as-measure* (Lobato & Thanheiser, 2002; Simon & Blume, 1994). In these cases it is difficult to develop an understanding that the ratio itself has real meaning in the situation and corresponds to something that can be perceived directly (e.g., steepness or speed) even though in many cases that ratio is difficult to measure directly and so is only quantifiable as a relationship between two other quantities (Lobato & Siebert, 2002). It may be challenging for students in the robotics context to be able to coordinate quantities of different measures, but doing so may have advantages in understanding and problem solving in which those ratios have consequential meaning.

A final math concept—and perhaps a more concrete rather than conceptual part of this analysis of the domain—is the particular strategies that students use which are evidence of understanding the proportional structure within robot movements. Although research has documented a wide range of proportional reasoning strategies (Tourniaire & Pulos, 1985), the most common valid strategies fall into two categories: *scalar* and *functional strategies*, which were described in the previous paragraph. Hoyles, Noss, and Pozzi (2001) showed how these different categories of solutions are both used by practicing nurses in place of formally taught strategies. In addition, they show how the use of the strategies is influenced by both the numerical structure of the problem but also by contextual factors specific to the workplace they studied, such as the ways different drugs tend to be packaged. This situated aspect to the solution

strategies suggests that there may be a lot to learn and understand about the particular situational factors that are involved in middle school students learning to connect proportional reasoning in problem solving about robot movements. As a result, a focus for the studies described here won't be just about the answers that students get, but also about their strategies for arriving at those answers and the extent to which they take advantage of the underlying proportional structure.

2.2.6   Research questions on environments for learning robots

The goal of the research studies in this part of the dissertation was to answer the following questions about the ways in which introductory robotics students connect the math of proportional reasoning with learning to control robot movements:

1. Do students connect math with their robot activities (focusing on the math of proportional reasoning and its connections to controlling robot movements)?

2. If they do connect math with robots, what is the nature and level of those connections?

3. What are the features of the learning environment that influence students' ability to engage with and make progress on making those connections?

4. What are the effects of the learning environment on learning and on engagement with respect to math in robotics?

2.3 STUDY 1 – SCRIPTED INQUIRY

Study 1 was an observational study focused specifically on a formal learning setting—a technology education classroom—that implemented a commercially available robotics unit. The formal learning setting is characterized by providing a considerable amount of structure to the learning environment. This is evident from a learning environment design perspective in that the goal of making the connection between math and robots is much more explicitly valued and targeted. The structure is also evident from a learning environment implementation perspective in the sense that students are placed in learning situations in which they are strongly guided to make those connections and so there will likely be many opportunities to observe those connections being made.

2.3.1 Activity context

The robotics unit that was analyzed for this study was designed for a wide range of students from upper elementary school grades to early high school grades as an introductory experience to learning robots in a structured, step-by-step manner. Because of the level of structure provided and because the activities were framed as "discovering" ideas, I will refer to this learning environment as the *Scripted Inquiry* environment. Although the focus for this study was on a classroom implementation with ninth- and tenth-graders, I have observed this same unit in other settings with younger students and found a similar implementation in terms of the organization of the activities and the types of student work that gets produced.

The designers of the unit were collaborators in the larger research program of which this dissertation project is a part. The unit designers were from a nationally recognized robotics education organization. Among other things, this organization develops curricula, leads teacher professional development sessions, and hosts robotics competitions. The director of the organization stated that the explicit purpose of this unit was to address, "technological literacy and mathematical competency using robotics as the organizer" (Email, January 31, 2008). In that sense, the unit had a very clear goal to teach students math with robots serving as the context.

All of the materials for the *Scripted Inquiry* unit were specifically designed to utilize the LEGO MINDSTORMS NXT platform and programming software (Figure 2 and Figure 3). The students built their robots using explicit instructions that would give them a common design that was tailored to the activities in the unit. For example, one activity has the students build an attachment that holds a marker right behind the wheel. When the robot makes a turning movement the marker traces the path that the wheel moved. Students are then able to measure the angle of movement as part of a unit activity on measuring turns. The robot was also designed to support gears so that gear ratios could be the focus of one of the lessons. Thus, a key aspect of the unit was to provide a carefully designed robot that could directly support the range of activities in the unit and the targeted concepts.
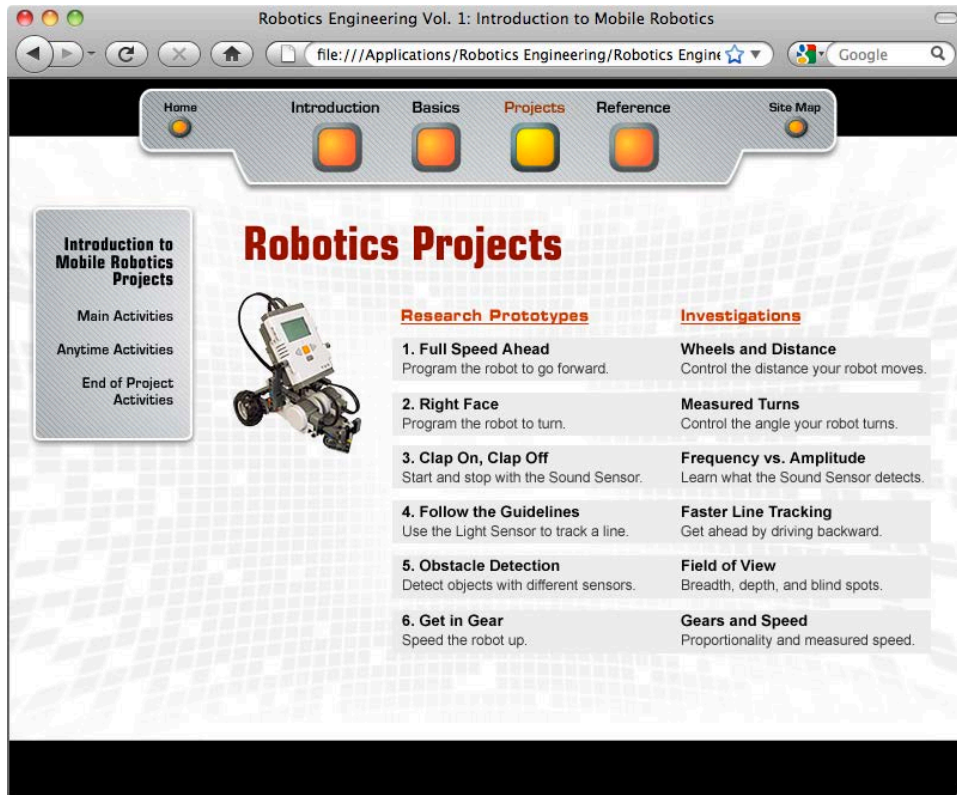
Figure 5. *Scripted Inquiry* sequence of activities

The *Scripted Inquiry* unit was organized around a set of multimedia lessons (text, animations, video, see Figure 5). The lessons were to be completed in order, alternating between a behavior programming module (e.g., program the robot to move forward, program the robot to follow a line using its light sensor, use gears to speed up the robot) and a related investigation of a STEM conceptual idea (e.g., the relationship of distance traveled to wheel size and number of wheel rotations, the proportionality between driving and driven gears in controlling the speed of a robot). In this sense, the unit was carefully sequenced so that students were explicitly guided toward building some behavior into the robot (e.g., creating and running a program to make the robot move straight forward) and then subsequently transitioned to a related investigation which

attempted to clarify and connect the math ideas that underlie that behavior (e.g., the distance a robot moves straight forward is equal to the number of motor rotations multiplied by the circumference of the wheel). The STEM investigation units were the activities in the unit that were intended to target the central mathematical concepts. For example, the "Wheels & Distance" investigation targeted the math concepts of diameter and circumference, ratios and proportions, means, and unit conversions (see Figure 6). The investigations also targeted science ideas, such as experimental design and error analysis, but the focus of this study was on the targeted math ideas.
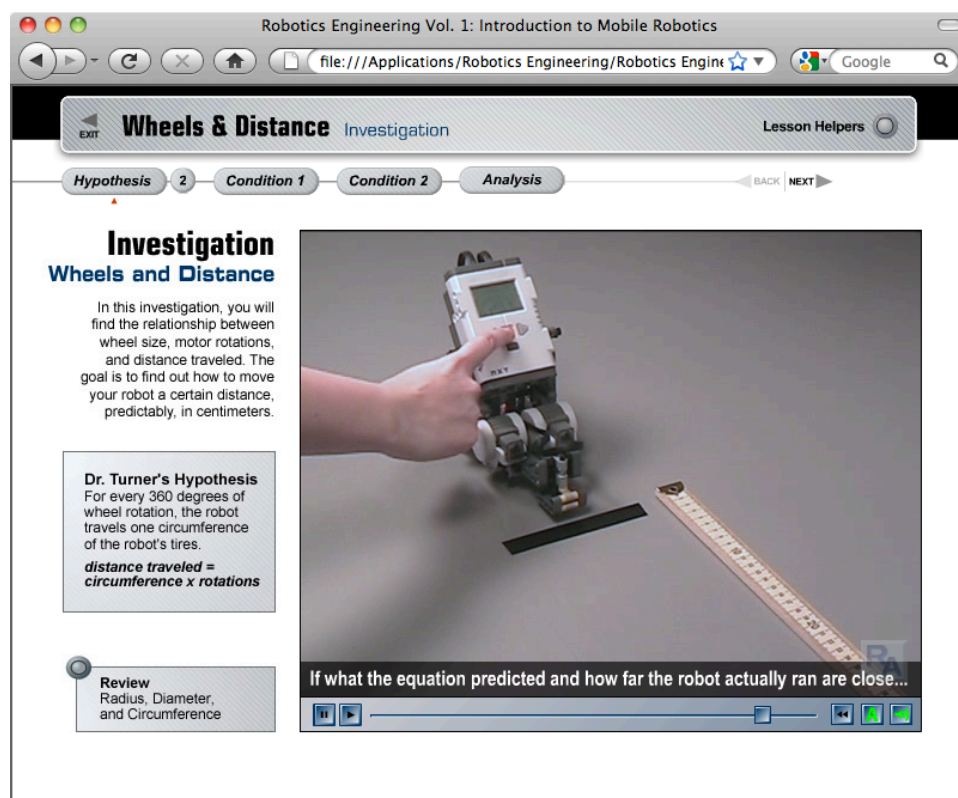
Figure 6. *Scripted Inquiry* "Wheels & Distance" investigation introduction screen

Only the first four lessons of the unit were observed for this study. The lessons included: (1) a building lesson involving learning how to program the robot to move straight; (2) an investigation involving learning how the size of the wheels is related to the distance a robot moves; (3) a building lesson involving learning how to program the robot to turn; and (4) an investigation on how to measure turns and control how far a robot turns based on fractions of the distance that the wheel has to travel to make the robot complete a full 360° turn. These four lessons were chosen because the unit designers highlighted these particular lessons as those that were best aligned with math concepts. The later lessons targeted other STEM concepts. These lessons specifically targeted ratios and proportions in the form of unit rates and equivalent fractions used both in the straight and turning investigations, but also connected to ideas of measurement and data analysis as well. Although each investigation lesson had a different story and targeted different concepts, all the lessons and activities in the *Scripted Inquiry* unit followed an approach in which students were given very explicit instructions at each step and asked to verify or test given relationships with provided representations and methodologies rather than generating their own.

The students completed the lessons in teams of two or three. Each team had one robot with which to work. In addition to the set of instructional screens viewed in a web browser that led students through the activities, each lesson was also supplemented by paper-and-pencil worksheets on which the students were to record their results and answer explicitly prompted questions. The typical and most common mode of interaction for students in the *Scripted Inquiry* environment was to be working with their team members around a shared computer terminal and a robot. They then followed along with the instructions in the lesson to complete the activities. On occasions when a unit task required them do so, they would run the robot on the floor or on a

nearby table, observe the results, answer questions on their worksheets, and then continue with the unit activities. The teacher served primarily in a helper role, facilitating only when students did not understand the instructions on the screen or were not able to implement the instructions properly.

For this implementation, the unit designers took active roles in the everyday activities. Although not part of the standard curriculum, the unit designers felt it was appropriate to provide supplemental activities to the existing multimedia resources for this implementation. The main additions and the justifications for each were: (1) add robot challenges that served to motivate and contextualize the problems that were the focus of the multimedia units; (2) add daily warm-up exercises of math problems both in robot and non-robot contexts to get students on-task from the start of class period and focused on thinking about mathematics generally; and (3) lead a number of whole-class discussions to introduce and then follow-up key activities in the unit lessons that they anticipated would be difficult for students to understand fully just from their teamwork and interaction with the standard unit activities.

An example of an added activity that was not part of the standard unit was the "Close Shave" challenge (Figure 7). This challenge was added as an introduction to the first pair of programming and investigation multimedia lessons that targeted moving straight distances. In this challenge, students were to create a set of programs so that their robot could move straight either 1, 2, or 3 floor tile lengths. The target number of floor tile lengths was randomly selected at run time for each team, a LEGO minifig was placed at the end of the selected number of floor tiles, and the team whose robot got the closest to the figure without tipping it over would win the challenge. In addition to setting the context for teams to start the "Wheels & Distance" investigation as a standard part of the unit, this challenge was also the focus of a follow-up

discussion about the results of the challenge intended to help students make some of the target math ideas explicit.

Figure 7. *Scripted Inquiry* "Close Shave" challenge description sheet

In sum, the *Scripted Inquiry* environment was highly structured in terms of the activities and steps that students engaged in when working with the robots, the order and connection of the activities with each other, and the explicit connections to math that were made as part of those activities. In addition, the tasks themselves were focused on building the capacity to program a simple behavior, and then following up that with an investigation focused on verifying some conceptual aspect of that behavior. But, supplemental challenges and whole-class discussions altered the features of the standard environment, providing a natural contrast to observe the resulting differences in how students connected the math ideas with those activities under those changed conditions.

2.3.2   Method

2.3.2.1   Participants

A total of 16 ninth- and tenth-grade students participated from 1 section of an introductory robotics course in an urban high school. The students worked in groups of 2 or 3 students per group, which resulted in a total of 7 groups.

The study took place within the context of an elective robotics magnet program within the school district. Because the program was open to all students throughout the district, the participants were from a wide geographic range of neighborhoods from the city. This was their introductory course for the program.

There were 20 total students in the class, but 4 students were excluded from the analyses because they did not complete the pre- and post-assessments.

2.3.2.2   Data sources

Problem solving assessment

Disciplinary learning is the first of the two key outcomes of learning environments evaluated within this framework. The *Problem Solving Assessment* was created to measure students' ability to solve quantitative problems in robotics and non-robotics contexts that aligned with the instructional goals of the unit. The items for the measure consisted of a combination of released items from the National Assessment of Educational Progress (NAEP) and isomorphic versions of those same items that were modified to use a robotics cover story. NAEP items were selected for inclusion in the measure that targeted concepts of proportional reasoning and concepts of measurement. Measurement items were included in addition to the proportional reasoning items based on the learning objectives associated with each of the prototypes and investigations in the unit. Eight items were selected for each content category and a corresponding eight items were created using a robotics context. Except for a few items that were grouped together, the items were then randomly ordered. Finally, two forms of the assessment each with sixteen items were created by alternating between the robotics and non-robotics form of each item except in the case of the grouped items. Each student was assigned randomly to one of the two forms at pre and the other form at post. The items used in both forms of this assessment are included in Appendix B.1.

Based on the sample in this study, each of the two assessment forms was adequately reliable (Cronbach's $\alpha = 0.78$ for Form A; Cronbach's $\alpha = 0.84$ for Form B). The forms were also adequately reliable within the content categories (Cronbach's $\alpha = 0.62$ for the measurement items in Form A; Cronbach's $\alpha = 0.67$ for the proportional reasoning items in Form A;

Cronbach's $\alpha = 0.77$ for the measurement items in Form B; Cronbach's $\alpha = 0.75$ for the proportional reasoning items in Form B).

Attitudes survey

Disciplinary engagement is the second of the two key outcomes of learning environments evaluated within this framework. Although disciplinary engagement may be measured in particular moments of activity (Engle & Conant, 2002), it is less clear how to measure it as the outcome of a whole unit or sequence of activities. In this series of studies, disciplinary engagement was operationalized as an individual student's ratings of their interest in the disciplines of robotics and math. In addition, because the focus of these studies was on facilitating students in making connections between these two disciplines, another aspect of disciplinary engagement was considered to be an individual student's ratings about the value of one discipline in service of the other, in this case, math in service of robotics. The *Attitudes Survey* was created to measure these two aspects of disciplinary engagement.

The engagement measure was adapted from validated scales of personal domain-specific interest in mathematics (Köller, Baumert, & Schnabel, 2001; Marsh, Trautwein, Ludtke, Köller, & Baumert, 2005) and of attitudes toward mathematics (ATMI, Tapia & Marsh, 2004). The version of the math domain-specific interest scale used in prior research consisted of five items for measuring interest or intrinsic motivation (Köller et al., 2001; Marsh et al., 2005). Three of those items were selected for this study focusing on different facets of interest and intrinsic motivation: "I enjoy working on mathematical problems" (affect), "I would even give up some of my spare time to learn new topics in mathematics" (self-determination), "While working on a mathematical problem, it sometimes happens that I don't notice time passing" (experiencing

flow). A fourth item from a separate inventory (Tapia & Marsh, 2004) was added in order to include an item that was negative on the construct: "Mathematics is dull and boring" (enjoyment). This item was reverse coded. These four items together made up the *math interest* (MI) subscale for this study. A parallel set of items was created by replacing the term "mathematics" with "robotics" in each of the items from the *math interest* subscale. The resulting four items made up the *robotics interest* (RI) subscale for this study.

To capture students' engagement in the connections between the disciplines of math and robotics, an additional subscale was created to measure students' attitudes about the relationship of math to robotics. The *value* subscale of the ATMI (Tapia & Marsh, 2004) was modified so that instead of measuring the students' beliefs about the usefulness, relevance, and worth of math in their life generally, it measured those same beliefs of math more specifically in robotics. Similar to the two interest subscales, three of the original items that were positive on the construct were selected. An additional item was modified so that it was negative on the construct and so was reverse coded. The phrase "in robotics" or "about robotics" was incorporated in each of the original items in place of more general phrases, such as "outside of school" or "in other areas." These four items made up the *math value for robotics* (MVR) subscale for this study.

In sum, the *Attitudes Survey* included three subscales: *robotics interest* (RI), *math interest* (MI), and *math value for robotics* (MVR). Each subscale contained 4 items for a total of 12 items. The survey was administered in a paper-and-pencil format. Students responded to each item on a 5-point Likert scale with response categories, $-2 = $ *Strongly Disagree*, $-1 = $ *Disagree*, $0 = $ *Neutral*, $1 = $ *Agree*, and $2 = $ *Strongly Agree*, such that positive values indicated a greater interest or value and negative values indicated a lower interest or value. Individual scores on

each of the attitude subscales were constructed by calculating the mean of the ratings on the items for that subscale. The items used in the *Attitudes Survey* are included in Appendix A.

Based on the sample in this study, the survey was adequately reliable both at the overall level (Cronbach's α = 0.80) and on the three subscales: *robotics interest* (Cronbach's α = 0.65), *math interest* (Cronbach's α = 0.81), and *math value for robotics* (Cronbach's α = 0.62).

Student work

Student work consisted of worksheets that followed along and supplemented the steps from the multimedia screen. Students worked in teams on completing the worksheets, writing down their answers to questions prompted by the multimedia screen, and showing their work when multiple steps were needed. Other work included additional worksheets that students completed documenting their strategies and results for challenges.

Observations and video of class activities

The author observed all whole-class and teamwork sessions. All of the whole-class sessions were video recorded. The video camera was stationary and focused on the front of the room where the instructor was leading the sessions. The unit designer (Mr. S) led all of the sessions as the primary instructor, but was occasionally assisted by the organization director who attended the whole-class sessions irregularly. The regular classroom teacher and the researcher participated primarily as observers. The whole-class sessions occurred both to introduce units and activities on which the students were about to start and to follow-up units and activities that they had completed. Although there were some explicit instructions and guidelines for these discussions in the teacher materials distributed with the unit, the focus of the discussions in this study were

adaptively chosen by the unit designer based on what was most relevant for the students at the time, especially when discussing the challenges. The structure and substance of these sessions varied considerably from one to the next. When the unit designer created some artifact in preparation for or in the course of the whole-class session, that artifact was also collected and used to supplement analysis of the video.

Interviews

Five students were interviewed after completing all of the activities in the robot unit. The interviews were semi-structured with a focus on understanding what about the unit activities were interesting to the students, their views about the connection between robots and math in the unit activities and more generally, and their sense of how to improve the unit activities. See Appendix C.1 for the list of questions. The students were chosen to reflect a range of achievement levels based on their performance on the post *Problem Solving Assessment*. Interviews were conducted with three higher-achieving students (two female and one male) and two lower-achieving students (both male). A lower-achieving female student was approached about participating, but elected not to participate in the interview part of the study.

2.3.2.3 Study design

Performance on the *Problem Solving Assessment* and responses on the *Attitudes Survey* were used as the dependent measures of disciplinary learning and disciplinary engagement respectively. The other data sources were used to identify the nature of the connections that students made between math and robots, and to identify the features of the learning environment's structure that influenced those connections.

2.3.2.4 Procedure

The class under study met every day of the week for forty minutes during the last class period of the day. The robotics unit was started on the first day of the second semester of the school year. In that first day, one of the unit designers made a presentation to the students introducing them to the unit and the pre *Attitudes Survey* was administered. Students were given 5 minutes to complete the *Attitudes Survey*. The students then spent the next several days building the robots that they would use for the unit activities. On the sixth day the pre *Problem Solving Assessment* was administered. Students were instructed to give an answer for every question, to show their work, and were permitted to use a calculator. They were given 30 minutes for the *Problem Solving Assessment*. The students then participated in the standard unit activities plus the additional activities supplemented by the unit designers, which included challenges, whole-class discussions, and warm-up problems. At the conclusion of the first two sets of behavior programming and STEM investigation units (after 30 class periods), the post assessments for both attitudes and problem solving were administered during the same class period. The students then continued to work through the remaining prototype and investigation activities from the standard unit, but with less direct intervention from the unit designers. Individual interviews with selected students (video recorded) were conducted after the students had completed all of the standard unit activities (after 50 class periods).

2.3.3 Results

2.3.3.1 Problem solving assessment

The first primary outcome of the study was students' changes in problem solving ability. The assessment included items targeting both proportional reasoning and measurement in both robotics and non-robotics contexts. Descriptive data from the *Problem Solving Assessment* are reported in Table 1. The data were analyzed using a multivariate repeated measures ANOVA with three dependent measures (proportion correct on the overall assessment, the proportional reasoning items, and the measurement items) and two within-subjects factors: time (pre, post) and context (non-robotics, robotics). For the overall multivariate ANOVA there was a marginally significant main effect of time, $F(3,13) = 2.59$, $p = 0.10$, $\eta^2 = 0.37$, but no significant main effect of context, $F(3,13) = 0.31$, $p = 0.82$, and no significant interaction between time and context, $F(3,13) = 0.84$, $p = 0.50$. This indicates that participation in the *Scripted Inquiry* unit may have had only a small positive effect on students' overall problem solving from pre to post.

Follow-up tests on each of the dependent measures revealed that there was a significant main effect of time on the measurement problems, $F(1,15) = 8.62$, $p = 0.01$, $\eta^2 = 0.37$, and a marginally significant effect on the overall assessment, $F(1,15) = 3.16$, $p = 0.10$, $\eta^2 = 0.17$, but no significant effect on the proportional reasoning problems, $F(1,15) = 1.01$, $p = 0.33$. Post-hoc comparisons using a Bonferroni adjustment indicate that mean difference from pre to post for the non-robotics measurement problems was significantly greater than zero ($M = 0.2$, 95% CI [0.03, 0.35]). The difference between pre and post for robotics measurement problems was not significant, nor were any other pairwise comparisons. This suggests that there were not enough evidence to conclude that students' experiences in the *Scripted Inquiry* unit had a positive impact

on their problem solving specifically in the robotics context or on their understanding specifically of proportional reasoning. However, the positive impact of the unit on students' problem solving in non-robotics measurement problems may indicate that the students learned more general skills related to measurement, but that those skills may not have been strongly-connected with their knowledge of robots.

Table 1. *Scripted Inquiry* problem solving outcomes results

Measure	Pre M (SD)	Post M (SD)	r	d
Overall				
Non-robotics	0.5 (0.2)	0.6 (0.3)	0.4	0.5
Robotics	0.5 (0.3)	0.6 (0.2)	0.6	0.2
Measurement				
Non-robotics	0.5 (0.2)	0.7 (0.3)	0.4	0.7*
Robotics	0.6 (0.3)	0.6 (0.3)	0.4	0.1
Proportional Reasoning				
Non-robotics	0.4 (0.2)	0.5 (0.4)	0.3	0.4
Robotics	0.4 (0.3)	0.5 (0.4)	0.5	0.1

[+] $p < .10$. * $p < .05$. ** $p < .01$. *** $p < .001$.

2.3.3.2   Attitudes survey

The second primary outcome of the study was students' changes in attitudes about robots and math. Descriptive data from the *Attitudes Survey* administered pre and post are reported in

Table 2. The data were analyzed using a multivariate repeated measures ANOVA with four dependent measures (average rating on the overall scale, and on the robotics interest, math interest, and math value for robotics subscales) and one within-subjects factor: time (pre, post). Although both at the overall level and on each subscale, students' attitudes changed negatively from pre to post, on the overall multivariate ANOVA there was no significant main effect of time, $F(4,12) = 1.05$, $p = 0.42$. A follow-up test revealed that for the *math interest* subscale there was a significant effect of time, $F(1,15) = 5.21$, $p = 0.04$, $\eta^2 = 0.26$, but there was not a significant effect on the overall scale, $F(1,15) = 2.28$, $p = 0.15$, or on either of the other subscales ($F(1,15) = 0.69$, $p = 0.42$ for the *robotics interest* subscale, and $F(1,15) = 0.17$, $p = 0.68$ for the *math value for robotics* subscale). This indicates that there was not enough evidence to suggest that participation in the *Scripted Inquiry* unit had a negative impact on students' level of engagement with robotics overall, but participation in the unit did have a significant detrimental impact on students' engagement with the discipline of math specifically.

Table 2. *Scripted Inquiry* attitudes outcomes results

Measure	Pre M (SD)	Post M (SD)	r	d
Overall	0.2 (0.6)	0.1 (0.6)	0.7	−0.3
Robotics Interest	0.0 (0.8)	−0.2 (0.9)	0.7	−0.2
Math Interest	0.4 (0.9)	0.1 (0.9)	0.8	−0.4*
Math Value for Robotics	0.3 (0.7)	0.3 (0.6)	0.5	−0.1

$^+ p < .10. * p < .05. ** p < .01. *** p < .001.$

2.3.3.3   Student work

Student work from the standard unit materials consisted of completed worksheets that were provided as supplements to the multimedia activities. In all cases, the work included records of numerical measurements and the results of calculations based on activities completed with the robots. In that sense, all of the work included some sort of mathematical activity. Only in rare cases did students include verbal explanations along with their measurements and calculations, and these were mostly limited to direct prompts from the worksheet rather than as explanations of a recorded numerical value. In many cases, the written work was exclusively the numerical result placed in the appropriate spaces on the worksheet. In some cases, students showed their work by also recording their operations when obtaining a calculated value. This provided evidence that all students participating in the activity did participate in some form of mathematical activity. However, most of that mathematical activity consisted of working with and recording numerical values (both measured and calculated).

Student teams did, however, also document some of their work in the additional challenge activities. For example, in the "Close Shave" challenge they were asked complete a worksheet in which they were to fill in the three duration values they used in their programs for each of the three distances, explain their strategy for finding their values, and then transfer their strategy to a new distance (7.5 tiles). Recall that this challenge activity occurred prior to engaging in the standard unit activities on straight distance. In this challenge activity, the solution strategies did vary substantively. One team used seconds as the units for their duration value, two other teams used rotations units, and the remaining two teams used degrees as their units. Their explanations of their strategies also varied and almost never included only numerical values or calculations. All five teams that completed the worksheet reported doing an initial

guess and then adjusting the value till they got the movement distance correct. Of those five, two of the teams reported using doubling and tripling from that correct value to get a value for the 2-tile and 3-tile distances (a scalar-based strategy). One of those two teams used that calculated value as their final value; whereas the other team adjusted that calculated value further by fine-tuning it until the results were satisfactory at each tile distance. A third team reported that they guessed "it took 1 second for each tile," and their results suggest they used that unit rate as the initial basis for their guesses for all three tile lengths, but then fine-tuned that value until the timing was right for each one. The responses to the transfer question were harder to classify, presumably since most of the teams guessed in the first place, and only the team that used a scalar strategy in the initial challenge was able to apply their strategy directly in this transfer problem. It is not surprising that many of the students used guessing initially given that this activity took place prior to the standard instruction units, but it is notable that the challenge was designed to make the patterns salient when scaling distance up from a base value, and three of the five teams did try to incorporate that aspect into their solution strategies at some level without being given such a strategy directly. In sum, for this more open-ended activity three of the five teams utilized some sort of mathematical activity based on scaling in their strategies.

2.3.3.4 Whole-class discussions

The whole-class discussions from the robotics unit did include lots of math. That is, the talk dealt directly with concepts such as percents, operations on decimal numbers, and statistics of central tendency, among many other math concepts. And each of those math concepts did follow directly from the activities of the unit. In the implementation for this study, however, some other important themes emerged about the substance of the whole-class discussions, their connections

to the activities of the unit, and the way in which they connect math and robots. I explore those themes with two contrasting whole-class discussions. The first discussion occurred right after the students completed the first STEM investigation—"Wheels & Distance"—as a way to review the main ideas for that unit activity. The second discussion occurred two days later in response to the "Close Shave" challenge that was completed prior to beginning the "Wheels & Distance" investigation. The winner of that challenge had not yet been announced, and so the unit designer took that as an opportunity to plan a discussion around how to determine which team was the closest team given that the teams were not all required to move the same distance. I describe each discussion in turn and then compare between them.

Post-investigation discussion

The goal of the "Wheels & Distance" investigation was to understand the relationship between wheel size, motor rotations, and distance traveled, and ultimately be able to use that information to make a robot move a given distance forward. The approach taken in the robotics unit was to have the students test a hypothesis equation that was generated by a fictional robotics researcher. Students were to calculate out theoretical values based on the hypothesis equation and test those values against actual values they measure from testing out on the robots. They were to use two different sets of wheels to see if the hypothesis equation worked for different wheel sizes.

Condition	Wheel Diameter (cm)	Wheel Circumference (cm)	Number of wheel rotations in program	Theoretical (predicted) distance traveled in program (cm)	Actual distance traveled (cm) in each trial	Average actual distance traveled (cm)	% Error
Standard Wheel	5.5 cm	5.5 * 3.14 = 17.27 cm	720 deg Or 720/360 Or 2	17.27*2 Or 34.54	1. 36 2. 35 3. 37	(36+35+37)/3 Or 36	(34.54-36)/34.54 Or 4.2 percent
Small Wheel	3 cm	3*3.14= 9.42 cm	2	18.84	1. 19.5 2. 20 3. 21	19.5+20+21=60.5 Or 20.17	18.84-20.17 = 1.33 7%
Small Wheel	7 cm	7 * 3.14 = 21.28	1.5 rot	21.28*1.5 = 31.92	30 32 33	95/3 = 31.6	(31.92-31.6)/31.92 1%

Figure 8. *Scripted Inquiry* post-investigation whole-class discussion data table

In this discussion, the students have completed the investigation activities, so the unit designer wants to review what they found and focus on percent error as a statistic for determining whether the actual and theoretical values are the same. An empty version of the data table in Figure 8 is projected onto a screen in the front of the classroom and the class fills out the cells together, the result of which is the completed data table in Figure 8. After completing all but the rightmost cell in the first data row, the unit designer introduces the problem as:

125. Mr. S: So, we said, we gotta check that value, that 34.54, the theoretical. And we got this measurement, 36, are they close enough?

126. Students: Yes.

127. Mr. S: How do you know?

128. Lance: Not that close, but…

129. Mr. S: What if it was 37, would they be close enough?

130. Lance: Yeh.

131.	Mr. S:	How about 38?
132.	Students:	No.
133.	Mr. S:	39?
134.	Students:	No.
135.	Mr. S:	38 and a half?
136.	Students:	No.
137.	Mr. S:	What about like thirty-nine point, or thirty-seven point nine?
138.	Students:	No … Yeh … Maybe
139.	Lance:	No, that's too close to 38.
140.	Mr. S:	Okay, so, the point I am trying to make here is there's not like a line, where we say, yes/no. It's actually kind of a grey area. The farther you get, the more you kind of go, uh [puts up hands weighing both sides]. And if it's real close, you know, then it's really obvious. Okay, so this thing called percent error is just there to tell us, how close are you. What percent is the difference? Like how different are they? Okay?

The unit designer used questioning to elicit the idea that the absolute difference between the theoretical values and the average of the actual values is not sufficient for determining whether the actual and theoretical are indeed different. He then proposes percent error as a better way to solve this issue. The unit designer then walks the students through a calculation of the percent error for the first row of data—the data on the robot with the standard wheel moving two motor rotations.

Together, the class calculates the percent error for that row as 4.2% using the calculations recorded in the rightmost cell of the first data row in Figure 8. Then the unit designer returns to the original question now framed around the percent error statistic:

| 141. | Mr. S: | Okay, so what is that number, it is 4.2%. That means that these two numbers, the theoretical and the actual are 4.2% different from each other. Is that a lot? Does that sound like a lot? |

The students give a range of "yes" and "no" responses, but most students do not take a position, and only one student articulates reasoning about the magnitude of percents:

| 142. | Tanisha: | Percents are a lot, but that's not a lot different. It's like in between. Oh, out of 100. |

So the unit designer decides that each student should vote. Presumably he did this to make sure that each student was engaged with the question, but it becomes clear in the ensuing discussion that not all students know how to make an informed choice. Lance articulates this reasoning:

143.	Lance:	It all depends on what you are talking about here. You know what I'm saying? It's not that easy.
144.	Mr. S:	It's 4.2. Okay.
145.	Tanisha:	4.2 out of 100.
146.	Mr. S:	If you think that they are not different, wait, did I just do that? Not different, raise your hand.
147.	Students:	[various responses]
148.	Lance:	Yeah, they are different, but understand like, it's hard to pick which one. I don't know.
149.	Mr. S:	And then, totally confused, but really paying attention?
150.	Tyrone:	Oh, I'm, yeh, right here.
151.	Kurt:	Yeh, that's what's up, right there.
152.	[Student]:	Totally confused. I just…
153.	Lance:	4.2 could be a lot in different cases.

| 154. | Mr. S: | Then let's look at this then. Let's do the next one, and we'll see. Maybe we'll find out 4.2 is a lot, maybe we'll find out it's not. What did you get when you measured the small wheels? You guys are going to guide this one instead of me. |

The unit designer recognizes the confusion and decides that moving onto the next row may alleviate some of the concerns by providing a point of comparison. So the class works through the calculations for the next row together—the data on the robot with the small wheels, also doing two motor rotations. They determine that the percent error for the small wheels is 7%, which they then agree is bigger than the 4.2% percent error for the big wheels. But this is a questionable conclusion to make given that even though the wheel sizes changed, the hypothesis equation was the same in both cases, so theoretically should have either been correct or incorrect in both cases as well. Instead of resolving this issue directly, the unit designer chose to move on to a set of "made-up" data on a 7-cm diameter wheel. The justification for this was most likely so that the students could have additional practice with the calculations and procedures for determining percent error, even though the interpretation of that value was still problematic. The central question of understanding the relationship between motor rotations, wheel size, and distance traveled was not addressed directly in the discussion.

Post-challenge discussion

This whole-class discussion occurred two days after the post-investigation whole-class discussion even though the actual challenge had occurred many days prior. The unit designer sets up this discussion by providing students a handout with each of the teams "Close Shave" challenge results including the team ID, the distance the team's robot had to travel (1, 2, or 3 tiles), the absolute difference between the robot's final position and the minifig, and a column

that he calculated in advance that divided the absolute difference by the number of tiles to get a

more standardized measure. The data table from the handout is included in Figure 9.

A Close Shave Results

Team	Tiles to Move	Final Distance to Minifig	% difference
Team 1	2	1.5cm	1.5 cm out of 60.96 cm = 2.46%
	3	2.0cm	2.0 cm out of 91.44 cm = 2.21%
Team 2	1	1.1cm	1.1cm out of 30.48 cm = 3.60%
Team 3	1	0.3cm	0.3 cm out of 30.48 cm = 0.98%
Team 4	2	3.5cm	3.5 cm out of 60.96 cm = 5.74%
Team 5	2	1.0cm	1.0cm out of 60.96 cm = 1.64%

Figure 9. *Scripted Inquiry* post-challenge whole-class discussion data table

The unit designer opens the discussion:

155.   Mr. S: Okay, so, you guys asked me yesterday who the winner was, and I went back to what we had recorded and I looked at it. I figured, well, who really is the winner here? Because what we said was, whoever gets closest to the man, right? Or the minifig. And then I took a look at one, how the numbers worked out again, and uh, somebody had brought up that it didn't seem fair that teams that just rolled an unlucky number or flipped an unlucky coin and got to go the really far one would have to come just as close to one that only had to go this far. So, what I did was, I took a look at the distance that you ended up from the figure versus how far you had to go. And I want you guys, actually, to pick the winner. That's why I took the team names off of these, so there's just numbers. So you tell me, who's actually closest?

The students shout out teams that they think should win mostly based on the absolute difference values (the "Final Distance to Minifig" column in Figure 9), so the unit designer tries to push them further by asking them to explain their reasoning and then getting them to attend to the different distances each team had to travel (the "Tiles to Move" column in Figure 9).

156.	Lance:	I mean, I don't know what you are saying.
157.	Mr. S:	Okay, okay, so, let's, if I gotta get close to this table, right? And it's right here versus it's down there. Did you ever play like golf, or something? You know, anything where you gotta get close to something. If it's really far away, it's hard to get closer, right?
158.	Darren:	Yeh.
159.	Mr. S:	If it's like right here, well, you should be able to get it in.
160.	Lance:	Yeh.
161.	Mr. S:	Same idea. How come the team that had to shoot three squares, should be graded just as far as the one that had to only go one?
162.	Lance:	Oh! So it ain't fair, nah.

The analogy with the golf game does seem to work in getting the students to attend to the different distances each team had to do, but the students don't immediately see a clear resolution and advocate instead redoing the challenge. Mr. S, the unit designer, tries to redirect their thinking to the "% difference" column in Figure 9 as an alternative way to judge teams across different distances:

163.	Tia:	No, he's saying why don't we do it over and give us all…
164.	Mr. S:	All the same one?
165.	Tia:	Yeh.
166.	Mr. S:	Okay, that might be a fair way to go about it, but I mean, we have already run it, and this is how, this is the data we got. This is the results. So, look at that very last column.

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167.	Tia:	You wouldn't be able to…
168.	Mr. S:	I don't know if you guys recognize it, but that's the same thing you had to do when we were comparing the di…, the way the robot went. We are just comparing how far off you were from the guy, divided by how far you had to go the whole total distance. So if your whole total distance was bigger, and you divided by a bigger number, then even if you had like three times as off, and you had three times as far, you got to [?] the same number. Does that sound fair? Or…
169.	Tanisha:	Well, why don't you divide them all by the same number?
170.	Mr. S:	Well, because they went different distances. Like the one that went one square, divided by one square. The ones that went two squares, divided by two squares. So they could be twice as far off at the end, but still work the same. Does that sound fair or not? Yeh.
171.	Tia:	No, it's not going to be fair any way you put it.
172.	Tanisha:	Well, why don't you divide then?
173.	Mr. S:	I did. That's what that last column is.

Although not all students followed the particular solution that the unit designer offered to standardize the differences, some students began to pick up on the same idea and offer their own solutions. For instance, Tia offered a multiplying solution instead of a dividing one:

174.	Mr. S:	Yes, okay, why Team 5?
175.	Tia:	Because they had to go further than Team 2 went. And if you would have multiplied the difference by 2, it would have been farther.
176.	Tanisha:	That's what I said. Why don't you just divide them?
177.	Mr. S:	Multiple the difference by 2? Why 2?
178.	Tanisha:	Because there's three tiles…
179.	Tia:	Because there's two [?]

180.	Mr. S:	Did you say multiply or divide? I am sorry, I didn't hear you.
181.	Tia:	Multiply.
182.	Mr. S:	Okay, so it should count like a 2.0cm off?
183.	Tia:	Huh?
184.	Mr. S:	Well you said multiply it by 2, so I multiplied 1cm by 2.
185.	Max:	Whose team won? Whose team won?
186.	Tia:	If you were to, alright if you were to take this and suggest like, okay, if they were to go 2 tiles, then it would be .6cm off. And then you multiply the difference.
187.	Mr. S:	Okay. Okay, I see what you are saying. So you are saying if you compare Team 3 and Team 5.
188.	Tia:	Yeah.
189.	Mr. S:	And you look at Team 3 and you say, well you should double that, because you know, if you scaled it up perfectly it would be twice as far and they would be twice as off. Is that what you are saying? And so the .6 is still less than the 1.0?

In this case, the unit designer understood what Tia was trying to do and so restated her idea so he could confirm it with her and make the reasoning more visible to the rest of the students in the class. Ultimately, the class does not come to a full resolution as to the winner and decides to take a vote, but the talk in the discussion suggested that many of the students were actively engaged in the idea of trying to find a way to fairly compare across teams that had to travel different distances. Furthermore, they began to use the ideas of scaling and relative amounts in their thinking about the situation. But, similar to the post-investigation discussion, this post-challenge discussion never addressed directly the central robot movement question of understanding the relationship between motor rotations, wheel size, and distance traveled.

Contrasting the discussions

Comparing and contrasting the two discussions, some important differences emerge that may have influenced the quality of engagement in the discussion. See Table 3 for a summary of the contrast between the two discussions, but I will discuss each in turn. One difference was that in the post-investigation discussion, the majority of the discussion happened over only one data point. Even though they eventually generated other data points, they served mainly as examples of the same theoretical prediction rather than comparing against a different theoretical prediction. This made the question of whether the percent error between the actual and predicted values was large or small a difficult one to answer. In contrast, in the post-challenge discussion there were five data points to compare from the beginning of the discussion, and all were directly relevant to the main problem of determining who should be declared the winner of the challenge. In addition, the unit designer re-represented the data so that students could focus on the most relevant aspects of the data to determine which group was actually closest relative to the distance they had to travel. Although the students still needed some assistance in attending to the target conceptual issue of relative differences, they did eventually do so and engaged productively in a discussion around that issue.

A second contrast was that in the post-challenge discussion, some time was spent just trying to understand the problem and question being asked. The unit designer had to improvise in providing appropriate analogies so that students could attend to the problematic parts that were connected most closely with the goals of the activity. That was not the case in the post-investigation discussion, where instead, they went through the activity in a procedural, step-by-step way. Related to this, students' seemed to understand the post-challenge problem not just in a numerical sense, but also in a situational sense as well. The issue that it wasn't fair for the teams

that had to move their robot further to be judged on the same absolute scale as teams that moved less far was something that the students' eventually understood well and responded to with reasonable solutions that had real meaning within that situation. The post-investigation solutions never achieved that level of meaning, and if anything, seemed to move further away as discussion went on and they practiced more iterations of the same calculations without making meaning of the whole activity and generating a solution to the central problem.

Table 3. Features of contrasting *Scripted Inquiry* whole-class discussions

Feature	Post-investigation	Post-challenge
Problem resources	One data point initially, later generated two other data points (one real, one fake), but all within the same theoretical hypothesis (varied on wheel size)	Six data points all based on actual data from team performance, at a range of distances, and in a clear table with only relevant data
Problem understanding	Did not directly respond to the "it depends" criticism or provide a way to judge when a percent error was large enough to justify "far" versus "close" (instead, provided more practice of the calculation of percent error)	Improvised analogies (golf) and worked with student questions to develop the problem
Problem engagement	Low level of engagement from students to determine whether fictional character's hypothesis matches with actual data	High level of engagement from students to determine which team deserved to win the challenge
Target disciplinary content	Percent error as a statistic for determining whether actual data supports a theoretical prediction	Percent error as a standardized way to judge deviation from a target for different distances

A third contrast may have been something that was determined prior to the discussion. That is, students' engagement in the problem at all. In the post-investigation discussion, the students' did not express any interest in determining whether Dr. Turner's hypothesis equation captured the actual movement data of the robot. The problem itself was not engaging. In contrast, in the post-challenge discussion students' were very interested to know which team had won the activity and so were in engaged in the overall problem from the start.

Despite these salient contrasts between the two discussions, what was particularly striking was that neither discussion centered on issues that were related to understanding robot movements in a direct way. The whole-class discussions were more focused on understanding variability of collected data—theoretical versus actual measurements, measured versus calculated values, percent error, etc. Very little attention was paid to using the actual physical context as a resource for thinking about the data—how the theoretical idea of predicting how far the robot would go as a function of the number of motor rotations was dependent upon the size of the wheels of the robot, or how the variability of distances traveled might be larger for larger distances and larger wheel sizes.

This non-physical-context focus is particularly salient in the post-challenge whole-class discussion. Recall that this challenge activity was added onto the original unit activities with the intention of providing an initial exposure to students to some of the proportional patterns, and a number of teams tried to reflect those patterns in their solution strategies. What was interesting, however, was that the discussion around the data collected did not focus on this proportional pattern between rotations and distance. Instead, it turned out to be further illustrative of the non-physical-context focus as the students tried to sort out which absolute difference would be the winner given that not all the teams had to do the same distances (a percent error problem as

modeled by the designer/teacher). They engaged in a debate in which the teacher helped them to understand the problem and then they argued for different teams as being the true winner. But the main idea that distance is related to the number of motor rotations or to the size of the robot's wheels was never addressed explicitly. It was also the case, that teams' actual strategies for solving the problem were not addressed explicitly, only the final results of their movements. In sum, although the post-challenge discussion was overall more engaging and the students presented more meaningful arguments in the discussion compared to the post-investigation discussion, neither discussion helped students focus on the core issues of understanding how to make the robot move specified straight distances.

2.3.3.5   Interviews

A couple themes emerged from the interviews with individual students that occurred after completing the entire robotics unit. The themes included: (1) a feeling that working with the robots to make them do things is preferable to working only on the computer, but that only making the robots do simple things was of limited interest even when connecting the math added challenge to the task; and (2) a sense that math does help with robots, but primarily because there are numbers involved (the program parameters and measurements) and so the math provides a more efficient route to getting the answer.

Making the robots do things is interesting, but doing overly simple things is not
Although exceptional, there was one case in which a student was just interested in the activities without qualification:

190.   Tanisha: I'm always interested.

191.	Mr. E:	Always? [Tanisha nods.] Okay. So why are you always interested?
192.	Tanisha:	Cause I love math. It's my favorite subject. And I like to work with robots. I like to build things. I like hands-on things.

Most of the interviewed students, however, were more particular about what sorts of activities they felt were interesting. For example, this student makes a distinction between doing problems only on the computer versus actually trying them out physically:

193.	Tia:	I feel like I did a lot more in middle school than I do in high school. A lot more hands-on stuff, and a lot more, like now, I don't know, everything's on the computer. Of course, you have to set up the robots and stuff on the computer. This class wise is okay, but my other classes that I have, we sit there. We do the computer the whole day. The whole period.
194.	Mr. E:	What are those classes? What are they called?
195.	Tia:	My Robotic Tech II class, we sit on the computer. This is Robotic Tech I.
196.	Mr. E:	So you don't work with robots?
197.	Tia:	In my other class? No. We're on a computer. We learn about series and parallel circuits. And things like that. And everything's on the computer. And I feel like, in that class, we can have way more hands-on stuff. Cause I learned about series, and the components, and the capacitors, and things, but I was working with them. Now you just do them on the computer. You sit there.

She expressed that in this unit, they did do a lot of hands-on work with the robots, and she felt that was positive. Tanisha and Tia were the two high-achieving females interviewed, both of who said that math was their favorite subject, so they may not have been representative of all the students in the class. Still, Kurt, a lower achieving male reaffirmed the idea that hands-on

activities are better than activities only on the computer by comparing to the activities they did in the same course during the first semester:

> 198.   Kurt: I'm always interested. I mean like, I like robots and stuff. Like, […mom…] when I'm home I'm always fixing stuff, taking stuff apart. I like robots and stuff. That's why I signed up for the program. But, like, at the beginning of the year, like I really didn't like it at all, because it wasn't really what it said it was. It said it was Robotics Tech, but we was doing nothing with robots. And then when y'all came, I was real interested and stuff, because we could like really work with robots and stuff.

But the other two male interviewees suggested even when working hands-on with robot that the particular types of robot activities mattered a lot. They both suggested that the work in this robotics unit was not as interesting as it might have been. Darren, a higher-achieving student made the distinction between working with robots doing simple tasks versus more complex tasks:

> 199.   Darren: To do more stuff with a robot. We could make it do different things, instead of just moving forwards and backwards.
>
> 200.   Mr. E: Like what?
>
> 201.   Darren: Like, noise level. Like, how to make it, um, follow your commands, and everything.

Lance, a lower-achieving male, articulated a similar idea:

> 202.   Lance: I mean, when they told me, okay, as I go older, stuff was going to get more challenging, do different things, LEGO robots was not nothing that came to my mind. That was something I felt like I should have did back then [in 8th grade].
>
> 203.   Mr. E: So you don't feel like it's challenging?

204.	Lance:	I mean it is challenging, but it's boring. This ain't interesting to me. Watching a robot, programming a robot to do something, that's not, I mean, it all depends. Like we're programming our robots to follow a light, this, this, and that, like, it's just boring. Like, I see if we did it every once in a while, but we're doing this, we've been doing this everyday for like a couple months now.
205.	Mr. E:	So you don't see programming this robot to do anything that's really?
206.	Lance:	No, it can do stuff, but it don't interest me, that's all.

Lance goes on to articulate that other activities that he did in middle school, such as building and racing CO_2 racecars and building and launching rockets, were more interesting to him than the particular tasks that they did in this robotics unit. One interpretation of both Darren and Lance's comments would be that the LEGO robots themselves were uninteresting, and there is certainly that element, but there is reason to believe that more complex tasks and challenges may have been sufficient to engage them. Overall, across the interviews there was a sense that working with robots in a hands-on way is engaging, but in order to sustain that engagement the types of tasks that you do with the robots have to be more complex than trying to understand just basic movement behaviors.

The role of math in robots is to provide an efficient route to the answer

Another theme that emerged from the interviews related to the students' views about the role of math in robotics. Since the interviews were only conducted subsequent to the unit activities it is not possible to determine whether these views existed prior to participating in the unit. However, some of their responses do suggest specific parts of the unit activities that influenced their ideas. In general, the students seemed to hold the belief that math is connected with robotics, and math can be very helpful in solving robot problems. For example, Tanisha articulated both how math

was used to measure and calculate aspects of the robot's design and behavior and that those

mathematical ideas worked well:

> 207. Tanisha: Like when we had to find, um, how long the distance, how much, how long, like, how far the robot would go. We had to use, like, calculate the rotations, and use pi, and stuff like that, toward the wheels. And like, most of the time when we did the project with, uh, our projects with, when the people made an educated guess, what do they call it? Hypothesis.
>
> 208. Mr. E: Sure, yeah.
>
> 209. Tanisha: They were usually right, when we use the math, when we do the math.
>
> 210. Mr. E: It worked well?
>
> 211. Tanisha: Uh-huh.

She then goes on:

> 212. Mr. E: Do you think [someone who knew robots, but not math] would not be as good as someone who knew math and robots? [Tanisha nods.] Yeah? So what would that person who knew both, what would, how would they be better?
>
> 213. Tanisha: Because they know exactly what they're doing. Like, if they have a problem, like, with math or anything, they can solve it easily. They figure out the problem in the robot.

Other students made the connection between math and robots as being primarily about working

with numbers. For example, Darren, a higher-achieving male responded to whether math is

helpful for doing robots by saying:

> 214. Darren: Yes. Cause you gotta get the threshold and everything. And it has numbers. And basically what math is is numbers.

Lance, a lower-achieving male, expressed a similar idea about the connections between math and

robots being about manipulating numbers, but then went further to suggest how the numbers

doesn't compel the math, but using math with the numbers distinguishes those who know how to

get answers compared to those who are just guessing. When asked if he could imagine someone

being good at robots, but not good at math, he responded:

215. Lance: No. That's just weird. I mean, there can be, but that's just weird. It's like the same thing, for real. You gotta use numbers with the robots.

216. Mr. E: Some people just put numbers in. Just try. Is that not doing math?

217. Lance: That's just guessing. You can guess forever. Instead of just doing the math. You gotta do the math to get the answer.

Tia, the other higher-achieving female expresses an idea about the relationship between math and

robots that is consistent with the rest of the interviewees. She articulates both that the connection

between math and robots is primarily about manipulating numbers, and that someone who didn't

know how to use math in the robot context successfully would be limited. When asked if math is

helpful for doing robotics, she replied:

218. Tia: It is, cause you have to know the degrees. You have to know things, yeah. And how to add and subtract. So, yeah.

219. Mr. E: Do you think someone could be good at robotics, but not very good at math?

220. Tia: You could, but not that good. You wouldn't go that far. No.

Tia reported that she thought of herself as being good at math but "okay" with robotics. But then

she went further to explain why she thought of herself as just okay at robotics. She replied that

robots includes math, but is not just math, as it also includes other stuff:

221. Tia: You have to memorize the programs, and the things, and the A wheel and the B wheel, and no. Uh-uh. That stuff will change, and it can change. And math is the same, so.

222. Mr. E: So, I'm trying to say, I'm trying to figure out what do you mean by the same?

223.	Tia:	Like, math, if you add one plus one, it's always going to be two. But with the wheels, if you add A and B, if you do the A wheel and the B wheel, it might do the same thing, but it will go a distance farther. Or, it'll, there's just different, it's too much. Too much. Then you have to learn different programs, and different, uh-uh.

Again, Tia's response further illustrates that math is seen as a resource for manipulating and working with the numerical values that are used with robots. But her elaborated response suggests that math isn't used to understand the way robots work directly, such as to be explicit about the relationships between aspects of the robot and its resulting behavior. Understanding robots from Tia's perspective seems to be about memorizing a large set of things without much in the way to unify them. Her lack of comfort with the "change" associated with robotics suggests that although the specific tasks that she had to do with robots in this unit were not difficult for her, she didn't grasp the underlying, more general ideas that may have consolidated some of that vast amount of stuff for her. In this sense, the connection of math to robots may have been limited for her. Given that Tia was a higher-achieving student, it is likely that other lower-achieving students and less-able math students in the class may have held similar ideas.

2.3.4 Discussion

In sum, the *Scripted Inquiry* approach had mixed results for helping students productively engage with and learn about how to control robot movements using math. In terms of learning, the students did make a significant improvement in their problem solving, but the improvement was not in proportional reasoning and was not in the robot context problems either. The qualitative data in the whole-class discussions also suggests that the math that the students were learning may not have been tightly aligned with their understandings (and misunderstandings) of

robot movements. Rather, whole-class discussions focused on other math ideas that didn't directly address how to understand the way the robots move, and so even when the discussions were productive in terms of aligning with some math idea, it would have been unlikely that they helped students in connecting the math to the robot situation specifically. Using one of those opportunities to compare and contrast the different strategies that the students used in the "Close Shave" challenge, some of which took into account the scalar nature of the problem, may have been better aligned and more productive for this purpose. Furthermore, students' belief that math is useful in robotics primarily for getting the "answer" more efficiently, suggests that a broadening of the role of math in their activities may help develop a fuller sense of the advantages of mathematics.

In terms of engagement, however, evidence from the *Attitudes Survey* suggests that students either made no change or were even more negative about robots and math after participating in the unit. The reflection interviews provided more support that the students felt that what they were doing with the robots was not that compelling. It may be that verifying some fictional character's abstract idea was not the best approach for making explicit a math idea for students to try out. Alternatives may include using math ideas not just in abstract verification experiments, but also as ways to solve actual design challenges or other sorts of activities that involve making the robot "do different things, instead of just moving forwards and backwards" (Darren's response on Transcript line 199). Although the capabilities of these particular robots are certainly limited, there may be possibilities for incorporating well-crafted robot challenges that make figuring out simple movements on the robots a more compelling option.

Overall, the structure of the *Scripted Inquiry* environment did help students use math in their robotics activities, but a stronger connection of the math to understanding and doing actual

robot movements may be a more appropriate focus for getting students to make progress on using proportional reasoning as a tool for understanding the movements of mobile robots.

2.4 STUDY 2 – DESIGN BASED

Study 2 built off the results from Study 1 in a design experiment that focused on the development and implementation of an alternative learning environment for helping students connect the math of proportional reasoning to learning to control simple robot movements. The results of the Study 1 suggested that the math could be more tightly connected to the robotics situation in the design of a learning environment, and so an attempt to do that was made in this alternative learning environment design.

Engineering design based learning (Kolodner et al., 2003; Sadler, Coyle, & Schwartz, 2000) provided a framework for the alternative learning environment design. Scripted inquiry environments (like the one in Study 1) are carefully crafted such that the tasks are clear and explicit in ways that students can follow along step-by-step as they build an understanding of an idea. Design based learning environments are also carefully crafted, but instead of being scripted, they are focused on providing scaffolding that allow students to take on as much of the cognitively challenging aspects of the situation as possible while minimizing peripheral aspects (Kolodner et al., 2003). For example, in the Electrical Alarm System design-based unit, which targeted electricity concepts, the designers purposefully removed the 9V batteries from the standard curriculum kits to force students to engage with the problem of how to combine 1.5V batteries such that enough power enters the system (Mehalik, Doppelt, & Schunn, 2008). Other

aspects of successful design based learning environments include having students create their own design based on a personal need to enhance personal engagement and breaking a complex task into subsystems so that students can build toward more sophisticated understanding (Mehalik et al., 2008). Of central importance is having a test against nature be how a student can assess their own success or failure as well as whether there is a need to revise (Sadler et al., 2000). The verification tasks used in the *Scripted Inquiry* environment were relatively abstract, which may have motivated the need for focusing on percent error as a target math concept for creating standardized measures that help comparing different data points, rather than proportional reasoning as a target math concept for understanding movements. But more importantly, a test against nature may prevent situations like what happened in the post-investigation discussion in the *Scripted Inquiry* environment when students had no basis for deciding whether the calculated percent errors indicated the theoretical and actual predictions were or were not the same. In sum, a *Design Based* learning environment may help students focus conceptually on proportional reasoning in robot movements by carefully crafting a task and providing resources that motivate the need for attending to proportional relationships in solving a design problem. Designing, implementing, and evaluating an environment for learning robots based around a design based framework would then provide a test of whether these alternative principles for carefully crafting the learning task would better help students engage with and learn about connecting math with robots.

2.4.1 Activity context

I developed the instructional activities and designed them specifically for this study. In order to better align the things that students do with the robots with the things that they ought to learn to better understand robotics, I designed the unit using a *Design Based* framework. After considerable brainstorming of potential design contexts that would be engaging to students and motivate them to consider the proportional relationships that underlie how the robots move, I decided to focus on robot dancing as the context. I designed the *Robot Synchronized Dancing* (RSD) unit with the primary objective of helping students connect proportional reasoning with how robots move and can be programmed to move in precise ways. All of the materials for the *Design Based* environment are included in Appendix D. The unit targets middle school students and highlights proportionality as a mathematical model for understanding how the physical characteristics of the robot, the parameters used in programming the robot, and the robots' actual movements are related. More specifically, the wheel circumference (distance around the wheels) and track width (distance between the wheels) are critical physical parameters that determine how many motor rotations are required to make the robot move a certain distance or turn a certain angle. As the wheel circumference increases, the amount the robot moves for each motor rotation increases in turn. By providing different robot types that vary on these physical parameters, students are setup with the opportunity to explore their effects. Furthermore, the goal of getting different sized robots to dance in sync with each other—do all the same moves at the same time—makes problematic the relationship between these aspects within an authentic design problem.

RSD is a careful blend of activities that is intended to be fun and accessible to young students while still being appropriately challenging. RSD attempts to capture students' experiences and interest in dance and in movements in general and connect it to the programming of basic robot movements. In the version of the RSD unit used in this study, the students take on the role of a knowledgeable dance choreographer who designs their own dance routine that they then program on robots provided to them. The students are provided with multiple robot "dancers" each with different physical characteristics (some wider, some narrower, some with smaller wheels, and some with bigger wheels) as part of dance team. Each team of students is assigned one of the robots and then creates their own dance routine set to a song of their choice from a list of available songs. Creating a dance routine was intended both to get students engaged in the overall robot task, but also to provide them with some general experience programming and adjusting robot movements so that they would have a basis for building more formal ideas later in the unit, similar to the "messing about" activities used in other design based learning environments (Kolodner et al., 2003).

In a short period of time, students are able to build creative and individualized dance routines for a single robot. They then begin to realize some of the difficulties in the task when they are challenged to use a second robot in order to develop a synchronized dance routine. That is where the conceptually challenging aspects of the task begin. In line with the tests against nature principle from design based learning environments research (Sadler et al., 2000), the differences between robots are perceptually salient to students and it is very clear in this situation whether the students have successfully synchronized the robots or not (Figure 10).
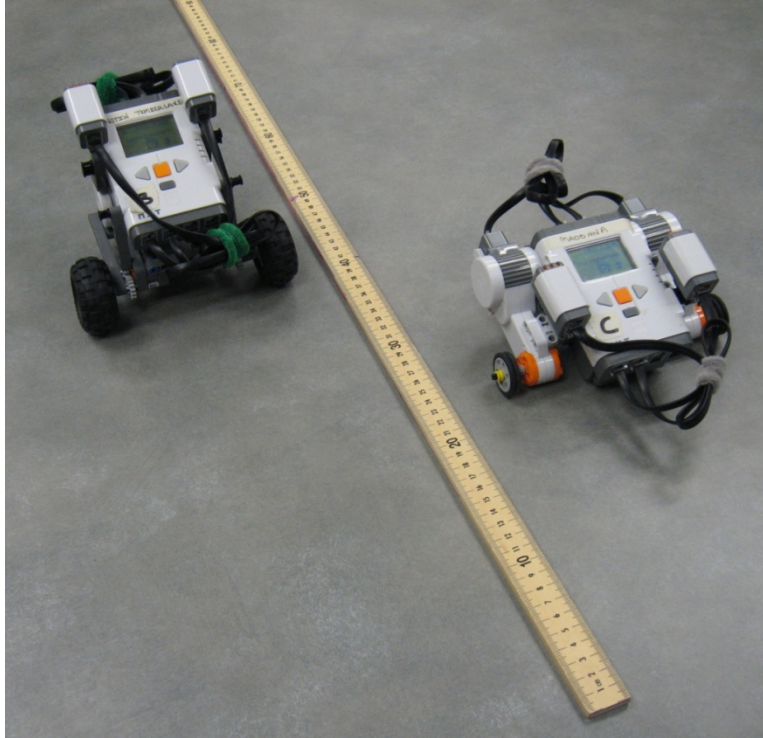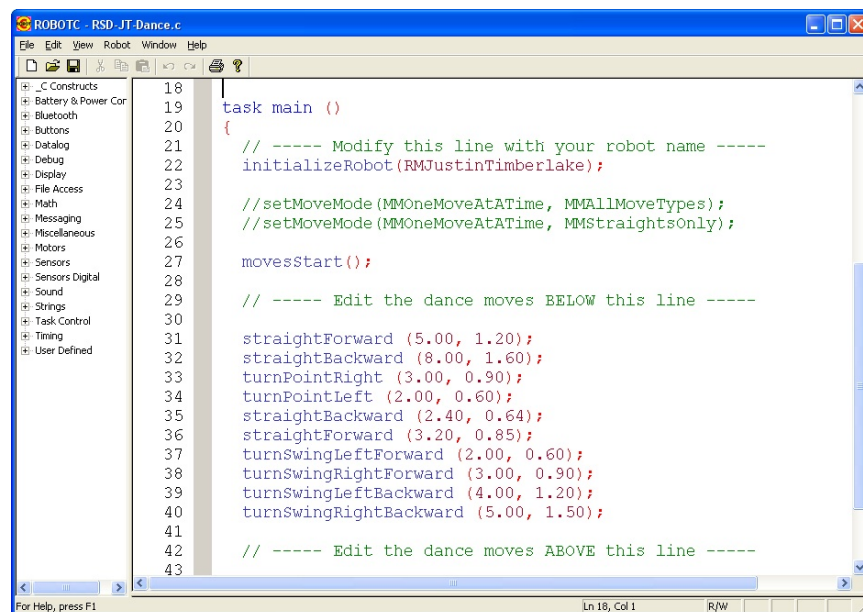
Figure 10. Two RSD robot dancers out of sync when using the same motor rotations

Other features of the learning environment were designed to provide students additional structure to help them attend to the synchronization task. For example, like in other design-based learning environments, the overall task was broken up into subtasks (or subsystems) so that students didn't have to address the whole problem at once (Apedoe, Reynolds, Ellefson, & Schunn, 2008). In the RSD unit, after beginning the synchronization aspect of the challenge, students would first work on synchronizing distance, and then work on synchronizing timing, and finally on synchronizing turns.

After designing their dance routine, the students created a design specification with precise measurements for each move in their dance routine so that they could use numerical values in addition to perceptual cues when trying to get the other robots to be in sync. Another

way in which the *Design Based* environment helped students attend to the quantitative aspects of the problem was by using an alternative text-based programming environment. The drag-and-drop programming interface commonly used when programming LEGO MINDSTORMS robots makes it difficult to attend to the numerical parameters (Figure 3). Instead, for the RSD unit students programmed using ROBOTC with functions that corresponded to each type of movement and were specially defined for the RSD unit (Figure 11). Also built in were functions to run the robot with stops between each move so that students could visually assess (and measure) whether that move was synchronized independent of other moves, and to only run certain types of moves (e.g., straights only) to support the subsystem task decomposition that is a feature of *Design Based* units.

Figure 11. A programming environment that makes the numerical parameters salient

In addition to these features of the environment that were set up as a standard part of the unit, the RSD context was designed such that it had a number of possible extensions that could deepen the context over time. For example, students might consider what would be required to modify if they wanted to enlarge or shrink their dance routine but maintain the overall timing. They might also consider robots that varied on other physical aspects, such as robots with varied gear ratios, which essentially changes the 1:1 relationship between motor rotations and wheel rotations. It was also believed that this text-based programming environment might ultimately lead to possibilities for using variables rather than just numerical values, as well as other ways of representing quantities in the situation and their relationships directly in the programming environment. That was not possible to do in the graphical programming environment.

The students worked in teams of two or three students on creating their dance routines, making a design specification, and attempting to synchronize their routine across robots. At each stage, students presented their work by demonstrating their robots. During the synchronization activities, teams also created posters that described their synchronization strategies and these were shared in whole-class discussions. The instructor tried to help students articulate their ideas verbally on paper and out loud, but all of the synchronization strategies were generated by the students.

In sum, the *Design Based* learning environment was substantively different than the *Scripted Inquiry* learning environment and attempted to address a number of the issues found when observing that environment. The prediction was that this alternative environment would better engage students by providing them an opportunity to use simple movements in a fun and creative way, while also helping them to better connect to the math of proportional reasoning

when inventing strategies for synchronizing their dance routine across robots of different physical dimensions.

2.4.2 Method

2.4.2.1 Participants

A total of 7 sixth- and seventh-grade female students participated from 1 group of an after-school program in a neighborhood urban middle school. The students worked in groups of 2 or 3 students per groups, which resulted in a total of 3 groups.

There were 3 sixth graders and 4 seventh graders. There were 4 African-American students (2 sixth, 2 seventh) and 3 Caucasian students (1 sixth, 2 seventh).

The study took place within the context of a community program targeting girls with the goal to encourage interest in Science, Technology, Engineering, and Mathematics (STEM) careers. Each of the participants volunteered to be a part of the STEM program and then to participate in the research study. All the participants attended the middle school in which the sessions were held.

There were 14 total students in the after-school program group, but 7 students were excluded from the analyses because they did not complete the pre- and post-assessments. All but one of the students who did not complete the program, dropped out prior to the activities on synchronization, and so those students did not experience any of the more mathematically rigorous activities intended to encourage the students to be more systematic and reflective about their conceptual understanding of the way the robots worked. Students stopped attending the program for a variety of reasons. Three of the students stopped attending after only the first two

sessions, while the teams were still building their dance routines. They indicated that the program was not what they had thought it would be or that they were unhappy with their team. Another three students stopped attending just after teams finished designing their dance routine and prior to any synchronization attempts. These students indicated that they could no longer attend because of an after-school conflict with a sports team. The remaining student who stopped attending did not share her reasons, but all the other team members in her group had stopped attending, so that may have played a part in her decision. Since all but one of these students dropped out of the program prior to any activities dealing with the synchronization across robots, it is unlikely that their attitudes toward mathematics and having to engage in rigorous activity was the primary motivator for their decision. For the remainder of this study, the analyses will focus only on the seven students who did complete the program.

2.4.2.2 Data sources

Problem solving assessment

The results from Study 1 helped to focus the *Design Based* unit on conceptual aspects of proportional reasoning. As a result, a new assessment was designed that was intended to better target assessment of a broad set of concepts that would indicate a deep understanding of proportional reasoning. Ten items were selected from published sources (Misailidou & Williams, 2003; National Assessment of Educational Progress, 2008). All the items targeted an abstracted understanding of proportionality. The items included a variety of contexts such as scaling, pricing, and speed, they included both verbal and table representations, and contained both direct and inversely proportional relationships. None of the items were set in a robotics context.

However, based on the sample in this study, the assessment was not adequately reliable (Cronbach's α = 0.44), and so inferential statistics were not conducted for this measure in this study. The items used in this assessment are included in Appendix B.2.

Attitudes survey

The *Attitudes Survey* was identical to the one used in the Study 1. Based on the sample in this study, the survey was adequately reliable both at the overall level (Cronbach's α = 0.73) and on two of the three subscales: *robotics interest* (Cronbach's α = 0.67) and *math value for robotics* (Cronbach's α = 0.81). However, the third subscale was not reliable, *math interest* (Cronbach's α = −0.21), and so inferential statistics were not conducted for this sample on that subscale.

Observations and student work

All the sessions were video recorded. The video data was used to document the design process for the student groups and to follow the progression of their synchronization ideas. In addition, the posters that teams created to share their synchronization strategies were used to assess the extent to which students connected proportional reasoning in the situation and the nature of those connections.

2.4.2.3   Study design

A teaching experiment (Steffe & Thompson, 2000) was conducted in order to better understand the "mathematical realities" of students with respect to their knowledge of how robot movements can be controlled. The teaching experiment allowed for viewing knowledge in transition as the

instructional experiences were intended to prompt students to make connections to mathematics but also to rethink some of their existing ideas. Disciplinary engagement and disciplinary learning were assessed both using the pre- and post- survey tools and using qualitative analyses of the video and poster data.

2.4.2.4 Procedure

The students met with the author after school two days a week in 1-hour sessions for a total of 26 hours. Participating students completed the problem solving and attitudes surveys on the first session and then on the last session of the program. Students were given 25 minutes for the *Problem Solving Assessment* and 5 minutes for the *Attitudes Survey*, one immediately following the other. They completed both surveys individually. On the *Problem Solving Assessment* they were instructed to give an answer for every question, to show their work, and were permitted to use a calculator. The first eleven sessions consisted of helping students to build their dance routines. It was not until the twelfth session that students worked on the synchronization problem. After that point, the activities were structured so that the students worked on given subparts of the synchronization problem and then regularly presented their solution methods to the rest of the group.

2.4.3 Results

2.4.3.1 Problem solving assessment

The students in this study improved only slightly from pre ($M = 0.3$, $SD = 0.2$) to post ($M = 0.4$, $SD = 0.2$) on the *Problem Solving Assessment*. However, as stated above, the low numbers of

participating students and the lack of internal consistency in the measure made a difference difficult to detect. Learning and the use of mathematics as a tool for problem solving can be assessed further using the qualitative data presented in the sections below.

Table 4. *Design Based* attitudes outcomes results

Measure	Pre M (SD)	Post M (SD)	r	d
Overall	0.6 (0.5)	0.9 (0.4)	0.6	0.7
Robotics Interest	0.9 (0.6)	1.0 (0.5)	0.5	0.1
Math Interest[a]	0.4 (0.6)	0.5 (0.6)	0.6	0.1
Math Value for Robotics	0.6 (0.6)	1.3 (0.5)	0.5	1.2*

[a] No inferential statistics were conducted on the *math interest* data due to the low level of internal consistency for that subscale in this sample.

[+] $p < .10$. * $p < .05$. ** $p < .01$. *** $p < .001$.

2.4.3.2 Attitudes survey

Descriptive data from the *Attitudes Survey* administered pre and post are reported in Table 4. Similar to Study 1, the data were analyzed using a multivariate repeated measures ANOVA. Both at the overall level and on each subscale, students' attitudes changed positively from pre to post. This was confirmed in the analysis as there was an overall significant main effect of time, $F(3,4) = 7.31$, $p = 0.04$, $\eta^2 = 0.85$. Follow-up tests revealed that the source of the significant effect was in the *math value for robotics* subscale, $F(1,6) = 9.85$, $p = 0.02$, $\eta^2 = 0.62$. There was not a significant effect on the overall scale, $F(1,6) = 3.25$, $p = 0.12$, or on either of the other

subscales ($F(1,6) = 0.14$, $p = 0.73$ for the *robotics interest* subscale, and $F(1,6) = 0.15$, $p = 0.72$ for the *math interest* subscale). This indicates that participation in the *Design Based* unit maintained students' overall engagement and interest, but also had a particularly strong positive influence on students' appreciation of the role of math in doing robotics.

2.4.3.3   Student work

Qualitative observations of the students indicated that they were very engaged at the start of the unit when they were creating their own dance routines. All the students were able to program the robots to do a variety of movements in time with the rhythm of the songs. However, creating the design specification, when they had to measure out each move, was much less engaging. The synchronization challenges were not as engaging as building the original dance but were more engaging than the design specification task.

When considering the nature of the students' ideas and the extent to which those ideas connected with the math of proportional reasoning two themes emerged.

Guess-and-check is not a unitary strategy and may have mathematical elements

Guess-and-check strategies dominated students' work in building their dance routines and also many of their attempts to do the synchronization as well. A theme that emerged from considering their work more carefully, though, was that the guess-and-check strategy contained various levels of mathematics within it. Guess-and-check, as a label for a strategy for working with robots or any other problem situation, actually refers to many distinct strategies with important conceptual variations between them, some of which may both be grounded in the situation and sophisticated mathematically (Nhouyvanisvong, 1999). For example, an initial strategy that

makes use of guess-and-check was described by one of the students when trying to get their

second robot to move the same distance as their first robot (see Figure 12):

224.	Mr. E:	So, tell us what you put on there and explain to us your strategy.
225.	Abigail:	Um, we used anonymous numbers, such as 9, 7, & 5. When we tried 5 it was close but not exact, then we tried 6 but it was too much, so we tried everything between 5 and 6, and when we tried 5.7 we got it.
226.	Olivia:	There's a lot of slashes on there.
227.	Sophia:	Doesn't anonymous mean unknown?
228.	Abigail:	Yeah, we used unknown numbers!
229.	Mr. E:	Maybe a good question to Abigail and Renee would be, ask them "what do you mean by anonymous? Can you explain what you mean by that?"
230.	Abigail:	Well we didn't try any particular numbers like half, we just tried any numbers.
231.	Olivia:	Random
232.	Sophia:	Yeah, that's random, anonymous is unknown.
233.	Abigail:	I said anonymous.
234.	Sophia:	Okay, okay, I'm just trying to help you out.
235.	Mr. E:	Ok, do you think random is a better.
236.	Abigail:	Yeah. Maybe.
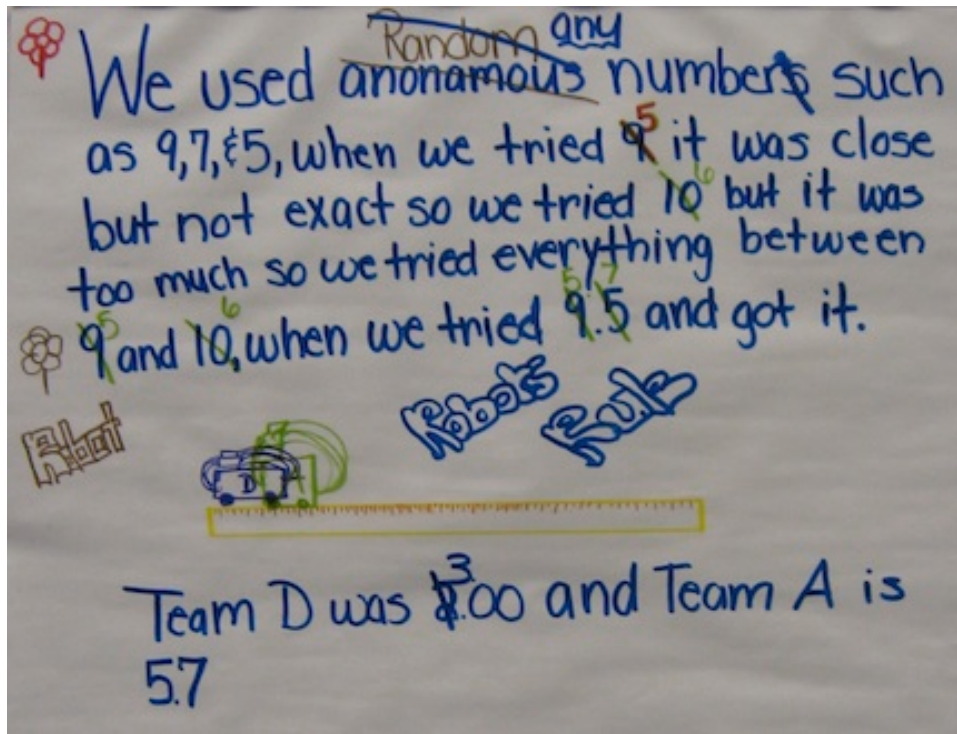
Figure 12. An initial guess-and-check strategy

Abigail's strategy does represent what may be commonly understood as a guess-and-check strategy, since she and her partner don't seem to have any particular way to determine what values to try. The next group to present had a different strategy in which they took the number of motor rotations that worked for a small-wheeled robot, then to get the number of rotations for a larger-wheeled robot, they divided by two and then subtracted an adjustment. Upon further questioning, it became apparent that their strategy also incorporated some aspects of guess-and-check:

> 237.   Sophia: Well, our strategy was dividing the number in half and subtracting a bit. If the wheels were bigger they take less time for rotations so that's how we really came up with that. And we think this works because the wheels on the second robot are a little less then half of the first robot.

238.	Mr. E:	So how is this different or the same or, um, then first strategy we heard, the random strategy?
239.	Sophia:	We didn't use random numbers.
240.	Mr. E:	Okay.
241.	Sophia:	We used… we used scientific numbers.
242.	Mr. E:	Scientific numbers? Okay, do you see any advantages or disadvantages to doing the random strategy or doing this one?
243.	Sophia:	I'd say a good advantage in doing this one, because this one is exact, and its scientific.
244.	Ava:	How much did you subtract it by, like how much? They said a bit.
245.	Mr. E:	Yeah, right, exactly. So a bit here, what is it? How much is "a bit" that's what Ava is asking.
246.	Sophia:	A small amount.
247.	Ava:	Like?
248.	Abigail:	Did you subtract .2?
249.	Ava:	3 or 4?
250.	Sophia:	Just, just to match it.
251.	Rebecca:	Just like, keep subtracting.
252.	Sophia:	Yeah.
253.	Rebecca:	Like, one each time until you get it.
254.	Sophia:	So we kind of in a way used part of the random strategy too, but we just took off half.
255.	Mr. E:	That's an interesting connection, Sophia I really like that. So maybe what they did is they cut off, they started doing something, what Sophia called scientific, by dividing in half, but then they had to do some random guessing to get it right, so there's maybe a little of an improvement over all random, but its not quite perfect. Yeah Abigail?

256.   Abigail: So all y'all did was divide by half and it was kinda close?

257.   Sophia: Yeah, and we just kinda had to take some off each time.

An important conceptual variation between these strategies is that the second group was both able to use the ratio in wheel sizes to make a good prediction, and then only had to use guess-and-check to account for an additional part that needed to be adjusted further to make the prediction more precise. The actual ratio of the wheel sizes (5.6/3.0) was close to half and so the halving strategy was a good approximation of the relative amounts that each robot moved given the same number of motor rotations. Using this halving strategy, and then recognizing that it needs to be adjusted further, seems to be an initial attempt by the students to recognize and incorporate the relevant proportional relationships into their synchronization attempts.

As described earlier, the "half-plus-adjustment" strategy does correspond to general proportional reasoning strategies that have been observed in the proportional reasoning literature. Misailidou and Williams (2003) observed an "incorrect build up" strategy that closely corresponds to this robot strategy. The "incorrect build up" strategy combines elements of addition and proportion, or as described in Tourniaire and Pulos (1985), "a multiplicative strategy on non-integer problems and then using a constant difference to handle the remainder" (p. 186). Thus, although the students did tend to use guess-and-check strategies, the different types of guess-and-check strategies that were observed may correspond to increasingly mathematical levels of understanding of the situation and could form the basis for a productive trajectory toward a more complete understanding.

One additional point, though, on Sophia's "divide-by-2-then-subtract-a-bit" strategy is that she actually got the numerical structure of the problem wrong in that the proper adjustment would have been to divide by two and then add a bit, not subtract. Unfortunately, in the setup of

this *Design Based* learning environment all of the students were trying to synchronize different movements and with different robots. Because of that, no other group had been working on the same problem with the same robots as Sophia, and that made it much less likely that someone else would hold her accountable to her idea. As the unit progressed, I made the choice to include some movements that all the teams would work on at the same time even though the particular moves were not in any of their dance routines. This provided some common basis of experience that helped to make the strategies that students invented more directly comparable when they were shared with the group.

Seeing and articulating the distinctions between strategies was difficult

Simply coming up with strategies that incorporated mathematics was difficult for the students. But another theme that emerged was that talking about the strategies, both in descriptive terms and in explanatory terms, was even more challenging. In many cases the students thought of their strategies as "guess-and-check" and labeled them like that or some close variant.

The following example shows Abigail again who started with the "anonamous" strategy (Figure 12). Abigail, who was the primary spokesperson for her two-person team, and her partner improved on their initial strategy and started to develop a more sophisticated strategy that used elements of scaling and building up to hone in on the correct value more quickly (Figure 13a):

258. Mr. E: Team D, if you could label your strategy for distance, what would you call it, guess and check or something else?

259. Abigail: Well I don't know because what we did was that two was the highest so we let it go at two, well, wait, wait … Yeah it was 2.0. We just tried 2.0 the regular number, and it went 40 centimeters, so we knew that if we did 4.0 it would go 80, so we added some and got 5.5. I'm not sure how to explain. It's just, just too hard

260. Mr. E: Okay, why don't we try to think of a name for yours. You don't think its quite guess and check, something a little bit different.

261. Abigail: Search and Find

In her case, although her strategy clearly used more sophisticated mathematical ideas, she did not have the language to express the mathematical operations that she did and that distinguished her strategy from other guess-and-check strategies presented prior to this point. Figure 13b shows another strategy developed later in the sessions. Again, the students who developed this strategy highlight the guess-and-check aspect of their solution by naming their strategy "Guess & Check" and rather than highlighting the unit rate that they used to relate motor rotations to distance.

(a)

(b)
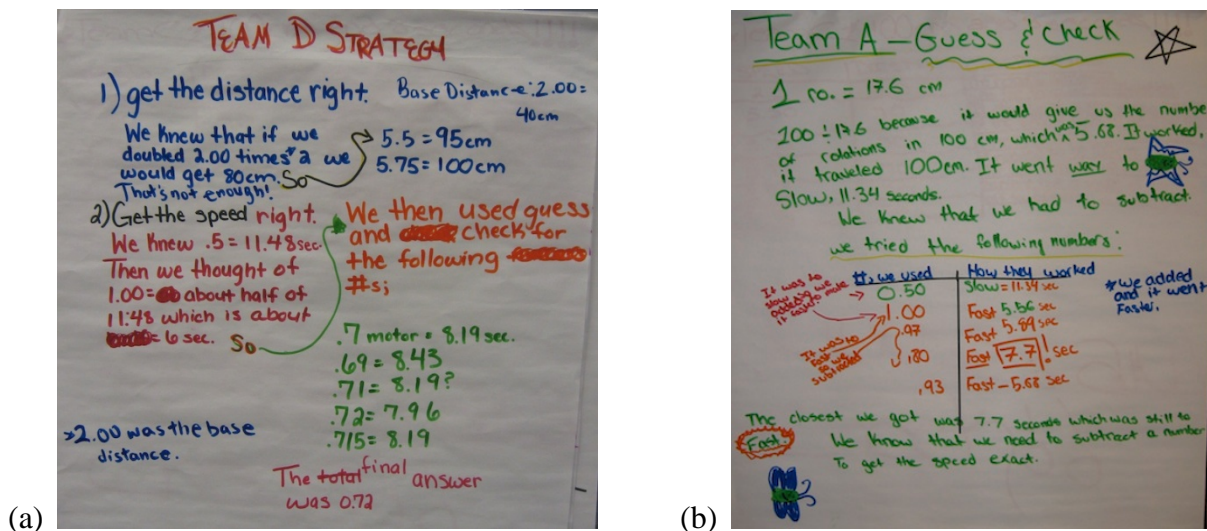
Figure 13. Two other strategies labeled as guess-and-check, but with other aspects

It is promising that within this *Design Based* learning environment the students were able to invent such a range of strategies that did include elements of proportional reasoning. On the other hand, it speaks to the challenge of moving beyond guess-and-check as a strategy to more formal and sophisticated methods. It may be that both Abigail and many of the others in the study could have benefited from some timely introduction of terms and strategies by the teacher. That is, it is possible that their experiences inventing their own strategies that attempted to capture and take advantage of the regularities that they observed informally may have prepared them to understand and appreciate some formal canonical strategies if presented to them (Schwartz & Bransford, 1998; Schwartz & Martin, 2004).

2.4.4   Discussion

In summary, the students' interest in mathematics and in robotics was maintained after completing a rigorous experience reflecting on their ideas about the way robots move in increasingly mathematical ways. In addition, their perception of the value of mathematics for robotics increased, suggesting that the experience in the *Design Based* learning environment provided opportunities for them to see how mathematics was relevant for understanding robot movements and for other more general aspects of robotics. Although the problem solving outcome measure was not reliable, the small pre to post gain suggests that these experiences may not have lead to large gains in general mathematics understanding of proportions outside of the robot context. However, the nature of their invented strategies did increase over time in the extent to which they incorporated aspects of proportional reasoning within robotics. This suggests that their experiences may have served as a foundation for adopting more formal

methods and possibly for transfer to other contexts if more explicit and timely scaffolding was included (Norton, 2006).

Unfortunately, initial pilot experiences with RSD suggest that although students' do get engaged in the synchronization task—and find it interestingly problematic—their understanding of the situation does not necessarily lead them to mathematize the situation right away and does not necessarily lead them to the level of sophistication of strategies that you would expect from a high level proportional reasoner. On the contrary, when left to their own devices, students focus intently on "making the robot do what I want" (Petre & Price, 2004), but their problem solving behavior is characterized mostly by guess-and-check strategies unless tasks in the learning environment are carefully crafted and implemented to encourage them to move beyond that. If they were left to continue, even within the RSD task it seems likely that they would create fine-tuned solutions to a particular dance routine or set of robots, but have to start from the beginning whenever given a new routine or new set of robots. This is clearly a suboptimal solution to the more general synchronization problem and so it is worth continuing to investigate how to improve on the *Design Based* learning environment and the RSD unit.

2.5   STUDY 3 – COMPETITION

Study 1 and Study 2 provided a solid basis for understanding how students might learn introductory robotics. But in both cases, the learning environments were somewhat formal with very explicit learning goals and lots of structured activities in which the students were expected to participate. A contrasting environment for learning robots from the more formal classroom

environments is the competition setting. Robot competitions are an increasingly popular way for students to get introduced to robotics. Students solve complex challenges that test their building and programming skills. Examining this context could significantly broaden the landscape of possible features of learning environments that target these skills. Similar to Study 1, I took an observational approach to examining this context in which I both assessed the learning and engagement of students who participated, documented the ideas and strategies that students used (especially with respect to connecting to mathematics), and identified the features of the learning environment that contributed to students' interaction within it.

2.5.1 Activity context

Robot competitions for grade-school-aged students involve the building and programming of small robots to solve a specific design challenge. As an example, the most popular annual robot competition is FIRST® LEGO® League (FLL), which uses the LEGO MINDSTORMS NXT robot platform. FLL is for students 9 to 14 years old (grades 3-8). A total of 14,725 teams from 56 countries participated in FLL in 2009—with up to 10 students per team (FIRST, 2010). FIRST also sponsors two competition programs for high school students that involve more complex robots, and one program for younger students (ages 6-9, grades K-3) called Junior FIRST LEGO League (Jr. FLL). The Jr. FLL league consists of a simplified version of the FLL challenge, but where teams present models or prototypes of their ideas to volunteer reviewers instead of in a competition setting. As a result, the FLL program is where students get their first, more substantive experience in building and programming robots to accomplish specific tasks, and so the focus of this study was at that level. Instead of an FLL competition, however, this

study was focused on a smaller, local-level robot competition, called "May Madness". May Madness was sponsored by the same robotics education organization that had developed the *Scripted Inquiry* robotics unit from Study 1. This organization also hosts the annual state FLL championship, but because FLL only takes place once a year in the fall, it hosts the May Madness competition to provide a smaller, more informal setting in which teams can compete during the spring season.

A robot competition challenge at this upper elementary and middle school level consists of a series of missions—each with their own designated point value—that involve pushing, retrieving, picking up, and placing objects around a 4' x 8' game board. The object is to earn as many points as possible in a limited time. The missions vary from competition to competition and change every year, but teams are generally given months to design their solutions. The challenge for the May Madness competition was adapted from another national-level competition program (Botball®). The particular challenge in this study was called "Botball Hybrid II", and like FLL competitions, it was to be completed with LEGO MINDSTORMS NXT robots and was geared toward elementary and middle school age students. The specifications of the challenge were announced nine weeks prior to the competition event, so teams had that entire time to prepare.

Although not quite as complex as typical FLL challenges in terms of the number of missions or the variety of objects on the board, the Botball Hybrid II challenge included a number of elements that required sophisticated solutions (see Figure 14). Two teams were to occupy the board at the same time, a black team and a white team. Each team could have one robot on the board at a time and the teams started at opposite ends of the board. The object was to get the most points possible in a 90-second round. Points were obtained by collecting ping-

pong balls and toilet paper tubes of the team's color and also common nests (small squares made from PVC pipe) and foam balls. Knocking the ping-pong balls loose gets some points, but the most points are obtained by bringing the objects back to a team's end zone. Even more points are obtained by lifting the objects into gutters on the side of the table.

Figure 14. *Competition* game board

2.5.2 Method

2.5.2.1 Participants

A total of 21 elementary and middle school students participated. Those students were from 4 teams that were labeled *Focus Teams* for this study. Each of the *Focus Teams* was from different

circumstances, including both school-based teams and community-based teams. Two of these *Focus Teams* were composed of middle school aged students and two of elementary school aged students. The *Focus Teams* were labeled with an "E" if they were an elementary-school-age team and an "M" if they were a middle-school-age team, followed by an integer to distinguish between the teams within the age groups: Team E1, Team E2, Team M1, and Team M2.

The students worked within their teams in a variety of configurations of groups of students, but all the analyses for this study were completed at the individual student level or at the team level. One of the elementary school aged *Focus Teams*, Team E1, consisted of only one student working on his own with the help of his father. Because he completed the pre- and post-assessments, his data were included in the analyses of change focused on the competition as a whole. But his data were excluded in analyses of change focused at the team level.

There were 28 total students among the *Focus Teams*, but 7 students were excluded from the analyses because they did not complete the pre- and post-assessments.

A total of 16 teams, including all of the *Focus Teams,* consented to participate in the part of the study that took place on the day of the competition. Of these teams, 9 were middle school age teams and 7 were elementary school age teams. Although most teams consisted of students in a mix of grade levels, the oldest team was made up of all eighth graders and the youngest team was made up of all second graders, so there was a fairly large range in grade levels. On average there were 7 students per team ($SD = 3$), with one team made up of a single student and three teams with the maximum of ten students. There were 22 total teams participating in the competition, of which 6 teams did not provide consent to participate in the study at all, so they were not interviewed about their solution strategy.

2.5.2.2   Data sources

Problem solving assessment

Because of the issues with reliability in the assessment used in Study 2, a new instrument was developed to measure students' ability to use math in robot problems. The revised *Problem Solving Assessment* (Appendix B.3) consisted of 10 multiple-choice and short-answer questions that asked the students to solve problems involving robot motion. The items were adapted from published sources of problems that assess aspects of proportional reasoning, and then were modified to focus on robot motion problems. Five missing value problems were selected from a diagnostic assessment of proportional reasoning (Misailidou & Williams, 2003). One of these five, the Mr. Short problem, was kept in its original form to serve as a transfer item. Two quantitative comparison problems were selected from research on classifying levels of reasoning in the balance scale task (Jansen & van der Maas, 2002). One item was adapted from a review of proportional reasoning research (Lamon, 2007) to assess the ability to distinguish proportional from non-proportional situations. Another item was adapted from research on modeling physical situations (Izsak, 2004) as a conceptual generalization problem. Finally, one more item was developed by the author to assess the ability to evaluate reasoning as being appropriate or inappropriate for a proportional situation. The overall assessment was adequately reliable for the sample in this study, Cronbach's $\alpha = 0.76$.

Attitudes survey

The *Attitudes Survey* was identical to the one used in the previous studies. Based on the sample in this study, the survey was adequately reliable both at the overall level (Cronbach's $\alpha = 0.87$)

and on the three subscales: *robotics interest* (Cronbach's α = 0.82), *math interest* (Cronbach's α = 0.82), and *math value for robotics* (Cronbach's α = 0.67).

Design strategy questionnaire

The *Design Strategy Questionnaire* (Appendix C.2) was created to gather descriptive information about each team and to assess the type of solution strategy that they used. It was a structured interview conducted by a researcher (the author and a collaborator) and included two parts. In the first part, the team's coach was interviewed about the number of students and adults on their team, their grade and experience levels, and the number of hours that the team met in preparation for the competition event. In the second part, one or two students from the team were asked to describe their solutions to the challenge and how they came up with those solutions.

2.5.2.3 Study design

Similar to Studies 1 and 2, the primary dependent measures included the change in robot problem solving as measured by the *Problem Solving Assessment* and the change in attitudes as measured by the *Attitudes Survey*, although these data were only available for a subset of the teams included in the study. The primary independent measure included the type of strategy that the team used in the design solution and these data were available for all of the teams participating in the study. Interviews using the *Design Strategy Questionnaire* assessed whether teams used math in their design solutions. An alternative outcome measure that was not available in the prior studies was teams' final rank in the competition, which was used as the dependent measure of engineering design success. Each team participated in three rounds, and their highest score of those three rounds was used to determine their final ranking in the competition. Further

data in interviewing the teams about their preparation activities and in analyzing the competition task provided insight as to which features of the learning environment contributed to the learning and engagement results as well as to the types of ideas and strategies teams used.

2.5.2.4 Procedure

The *Problem Solving Assessment* and *Attitudes Survey* were administered to the students on the *Focus Teams* soon after the competition scenario was released to provide an assessment prior to the majority of the teams' preparation activities. The competition scenario was released 9 weeks prior to the competition event and all of the *Focus Teams* were surveyed within 4 weeks of that time. The surveys were administered at each team's normal meeting location. Students were given 25 minutes for the *Problem Solving Assessment* and 5 minutes for the *Attitudes Survey*, one immediately following the other. They completed both surveys individually. On the *Problem Solving Assessment* they were instructed to give an answer for every question, to show their work, and were permitted to use a calculator. On the day of the competition, researchers interviewed all of the teams participating in the study using the *Design Strategy Questionnaire*. Finally, in the weeks immediately following the competition, the *Problem Solving Assessment* and *Attitudes Survey* were administered a second time to the students on the *Focus Teams*. The paper-and-pencil instruments were unchanged from the previous administration, they were administered again at each team's normal meeting location, and the procedure was identical.

Table 5. *Competition* problem solving outcomes results

Team	n	Pre M (SD)	Post M (SD)	r	d
All Teams	21	0.6 (0.3)	0.7 (0.3)	0.8	0.3*
Team E2	3	0.3 (0.3)	0.4 (0.2)	0.9	0.8
Team M1	8	0.6 (0.3)	0.6 (0.3)	0.8	0.0
Team M2	9	0.7 (0.1)	0.8 (0.1)	0.4	1.1+

+ $p < .10$. * $p < .05$. ** $p < .01$. *** $p < .001$.

2.5.3   Results

2.5.3.1   Problem solving assessment

Descriptive data from the *Problem Solving Assessment* are reported in Table 5. A paired *t*-test revealed there was a significant increase in students' overall problem solving from pre to post ($t(20) = 2.47$, $p = 0.02$, Cohen's $d = 0.3$). This indicates that participation in the *Competition* did have some positive impact on students' overall problem solving. Closer inspection suggested that there might have been large differences in outcomes between the teams. To examine whether this was true, the data were analyzed using a repeated measures ANOVA with the proportion correct on the problem solving assessment as the dependent measure, time (pre, post) as a within-subjects factor, and team (E2, M1, M2) as a between-subjects factor. Using this model, there was only a marginally significant main effect of time, $F(1,17) = 3.31$, $p = 0.09$, $\eta^2 = 0.16$, but there was a significant main effect of team, $F(1,17) = 5.12$, $p = 0.02$, $\eta^2 = 0.38$. The interaction term was not significant, $F(2,17) = 2.04$, $p = 0.16$. This suggests that there were some differences

between the teams, and that there also may have been some improvement overall from pre to post.

Follow-up tests on the main effect of time using a Bonferroni correction adjusting for multiple comparisons suggest that there was a marginally significant mean difference from pre to post for Team M2, the second middle-school-age team ($M = 0.10$, 95% CI [−0.01, 0.20], $F(1,17) = 3.96$, $p = 0.06$), but not for the elementary-school-age team, Team E2 ($M = 0.13$, 95% CI [−0.05, 0.32], $F(1,17) = 2.40$, $p = 0.14$), or the first middle-school-age team, Team M1 ($M = 0.03$, 95% CI [−0.08, 0.14], $F(1,17) = 0.28$, $p = 0.61$). This suggests that there may have been some improvement made by Team M2, but the low number of students from Team E2 may have made a statistically significant improvement difficult to detect for that team.

Follow-up tests on the differences between teams suggest that both middle-school-age teams were significantly greater than the elementary-school-age team, Team E2, at pre test (difference of Team M1 to Team E2, $M = 0.37$, 95% CI [−0.03, 0.78], $p = 0.08$; difference of Team M2 to Team E2, $M = 0.46$, 95% CI [0.06, 0.86], $p = 0.02$), but the only differences at post test were that Team M2 was significantly greater than Team E2 ($M = 0.43$, 95% CI [0.04, 0.82], $p = 0.03$). The pattern of results at pre test could be explained by the elementary-school-age students having less background in mathematics compared to the middle-school-age students. The pattern of results at post test could be explained by Team M2 continuing to make improvements in their problem solving even though they started at pre test with very high scores on the problem solving assessment. On the other hand, the lack of pre-post change for Team M1 may mean that they did not make progress and could suggest that the differences between Team M1 and Team E2 became less over time as a result of participation in the competition.

2.5.3.2 Attitudes survey

Descriptive data from the *Attitudes Survey* administered pre and post are reported in Table 6. Students' attitudes changed negatively on the *robotics interest* subscale and positively on the *value of math for robotics* subscale. Similar to Study 1, the data were analyzed using a multivariate repeated measures ANOVA to investigate whether those changes were significant at the level of the whole competition. Time was a marginally significant main effect across all of the measures, $F(3,17) = 2.78$, $p = 0.07$, $\eta^2 = 0.33$, but time was not a significant effect on any of the measures individually. This indicates that was not evidence to support the conclusion that participation in the *Competition* impacted students' attitudes.

Similar to the *Problem Solving Assessment*, there may have been differences in changes in attitudes between the three *Focus Teams*. So a follow-up analysis was done using a multivariate repeated measures ANOVA with the proportion correct on each of the *Attitudes Survey* subscales and the overall scale as the dependent measures, time (pre, post) as a within-subjects factor, and team (E2, M1, M2) as a between-subjects factor. In this analysis there were no significant main effects or interactions both in the overall multivariate test and in the univariate tests as well. This suggests that there was not enough evidence to conclude that the *Competition* environment impacted students' engagement with robotics and math, and that there was not enough evidence to suggest differences in engagement between the teams.

Table 6. *Competition* attitudes outcomes results

Measure	Pre M (SD)	Post M (SD)	r	d
All Teams				
Overall	0.7 (0.7)	0.7 (0.9)	0.6	0.0
Robotics Interest	0.9 (0.8)	0.8 (1.1)	0.5	−0.2
Math Interest	0.3 (1.0)	0.3 (1.2)	0.7	−0.1
Math Value for Robotics	0.7 (0.7)	1.0 (0.8)	0.4	0.3
Team E2				
Overall	0.4 (0.5)	1.1 (0.9)	1.0	1.1
Robotics Interest	0.6 (0.1)	1.1 (0.7)	0.5	1.2
Math Interest	0.0 (1.1)	0.7 (1.4)	1.0	0.6
Math Value for Robotics	0.8 (0.3)	1.4 (0.5)	0.2	2.0
Team M1				
Overall	0.2 (0.7)	0.2 (0.7)	0.9	0.0
Robotics Interest	0.6 (1.0)	0.5 (1.2)	1.0	−0.1
Math Interest	−0.1 (1.2)	−0.3 (0.8)	0.8	−0.3
Math Value for Robotics	0.2 (0.7)	0.5 (0.6)	0.6	0.4
Team M2				
Overall	1.0 (0.4)	0.9 (1.0)	0.0	−0.2
Robotics Interest	1.2 (0.6)	0.8 (1.2)	0.0	−0.4
Math Interest	0.8 (0.8)	0.6 (1.3)	0.7	−0.1
Math Value for Robotics	1.1 (0.5)	1.3 (0.9)	−0.3	0.2

[+] $p < .10$. * $p < .05$. ** $p < .01$. *** $p < .001$.

2.5.3.3 Student work

The different strategies

The teams came up with a range of qualitatively different solutions. This range included choosing which parts of the challenge to pursue and in what order. However, every solution strategy included at least one common component—moving the robot to the center of the board to begin scoring points. I focused on the solution strategy for this component in order to compare across teams. This was also effectively the same basic movement problem that was the focus of the learning environments in Study 1 and Study 2. Table 7 is a list and description of the different solution strategies that teams used, and the number of teams that used each approach out of the 16 teams that were interviewed on the day of the competition.

That only 3 teams used a (non-rotation) *Sensor-Based* strategy is likely a direct consequence of the nature of the particular robot challenge. In particular, the toilet paper tubes were not steady enough for a robot's touch sensor to contact them without tipping the tubes over. As a result, teams seeking to score using the tubes had to choose non-contact means of controlling their robot's movement. The 3 teams that did use a *Sensor-Based* strategy on their first move were all going for the nests, which are much heavier than the toilet paper tubes. However, for various reasons, even these teams abandoned use of their sensors in their moves later in the challenge. In addition, the board surface featured few marked lines, making line-following and line-tracking less attractive.

Table 7. *Competition* observed strategies

Strategy	*n*	Description
Guess-Test-Adjust	6	Students guess an initial value for the motor rotations, try it out on the robot, and then adjust the value to be bigger or smaller based on whether the robot went too far or not far enough. It is often not clear how students arrived at their initial guess. Teams who used this strategy also differed in how they made the adjustments: some used a systematic strategy in which they went up by whole numbers first, then smaller numbers, while others used more arbitrary adjustments.
Calculate-Test-Adjust	4	The only strategy that was explicitly math-based. Students measure the distance the robot has to move. They then make a mathematical prediction about the correct rotation value for the movement based on the size of the robot's wheels or a known distance the robot moves in one rotation. All of the teams who made their initial calculation this way had to fine-tune that value afterwards using adjustments that resemble the *Guess-Test-Adjust* strategy or the *View-Mode* strategy.
View-Mode	3	Students use the view mode on the NXT and then "walk" their robot (push it by hand as the wheels roll along the ground) to the desired destination. They read the value displayed and use that value in their program.
Sensor-Based	3	The only strategy in which the robot does not travel a set number of wheel rotations (or duration of time). Students program the robot to move until a physical sensor stimulus provides a cue to stop. For example, running forward until the robot bumps into a nest and a Touch Sensor is triggered.

The remaining 13 teams programmed their initial move using the rotation sensor, effectively moving a set distance forward. However, those 13 teams used qualitatively different methods to choose their motor rotation values, especially the initial value. Some teams guessed; others used the view mode; but 4 teams chose to start with a math-based prediction based on a

measurement of the desired robot movement. Although certainly not a majority of teams, 25% of the teams interviewed did use mathematics explicitly in their design solution in this way.

A math-based strategy for calculating motor rotations

Teams making math-based predictions used several different mathematical relationships to arrive at their predictions. For example, one group measured how far the robot moved forward with each motor rotation, and then calculated how many of those 1-motor-rotation distances the robot needed to move the total distance to the target. The students then entered this value into their program, tested it, and fine-tuned the value to get the robot to exactly the right spot. One notable quality of this strategy is that it is not purely mathematical—all 4 teams that used *Calculate-Test-Adjust* for their initial motor rotations value ended up having to refine their value with guessing or with the view mode afterward. A math-based measurement and prediction does not appear to be sufficient on its own for this type of competition challenge.

The relative success of the different strategies

The ranks of the teams who used each strategy were compared to assess the extent to which each strategy was related to design success. Figure 15 shows the distribution of ranks in the competition of the teams who used each strategy. Inspecting the mean ranks, the *View-Mode* strategy was on average the most effective and the *Sensor-Based* strategy was on average the least effective, and this particular contrast was statistically significant ($t(4) = 4.98$, $p = 0.01$). The *Guess-Test-Adjust* and the *Calculate-Test-Adjust* strategies were in the middle and similar to each other. As stated above, it may have been that this particular challenge was somewhat biased

against the use of sensors, and so it is not surprising that teams who used the *Sensor-Based* strategy did not perform well.
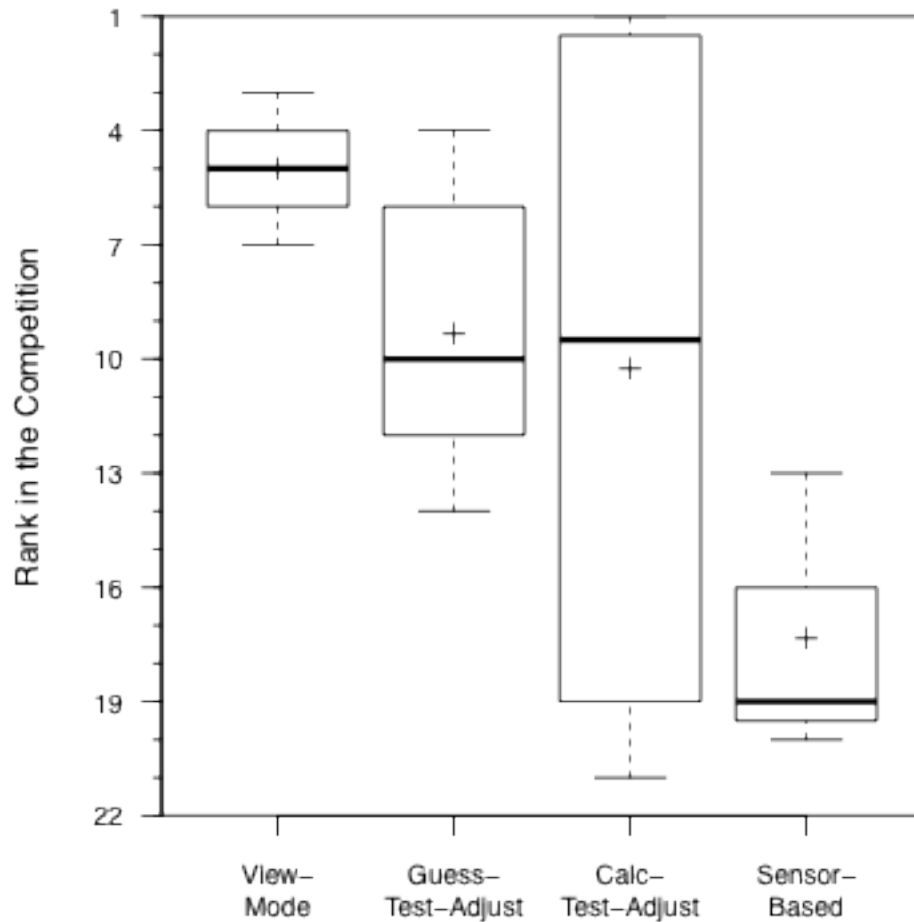
Figure 15. Distribution and mean (+) of ranks in the *Competition* based on strategy used

The teams using the *View-Mode* strategy did perform particularly well. I hypothesize that this strategy may lead to success for two reasons. First, teams that use this strategy can program their movements quickly. Figuring out the correct motor rotations value is straightforward and fast, so that frees the team up to spend their limited time improving other parts of their solution

(e.g., making their robot base solid and their attachments functional). Second, the *View-Mode* strategy is very reliable, so once teams get a motor rotation value by using this strategy, they then have a lot of confidence that that value is the right one and will work well. In essence, the *View-Mode* strategy is easy to implement quickly and gives very reliable results, which explains why teams who chose that strategy tended to do well in the competition and there was very little variance among them.

The success of the math-based strategy

Compared to rolling the robot on the ground and reading a number, both *Guess-Test-Adjust* and *Calculate-Test-Adjust* are slow to implement and potentially less reliable as well. Again inspecting Figure 15, on average teams who used these strategies performed at an average level in the competition, and not as well as teams who used the *View-Mode* strategy. Alternatively however, when inspecting the variation within strategies, the *View-Mode* strategy was the least variable, but the *Calculate-Test-Adjust* strategy has a large variability spanning almost the entire range of possible ranks.

A closer look at the 4 teams that used the *Calculate-Test-Adjust* strategy shows that two of them were the top ranked teams in the entire competition (ranked #1 and #2 out of 22 teams). This suggests that using a math-based measure-and-predict strategy can be very powerful. At the same time, the other two *Calculate-Test-Adjust* teams were ranked #17 and #21—the complete opposite end of the spectrum in terms of design success. Figure 16 illustrates this more clearly, showing each team as an individual point and examining their max score in the competition, which is what determines their final rank. In this figure, the bi-modal distribution within the *Calculate-Test-Adjust* strategy is evident, as well as the relative success in terms of absolute

score of those teams that did implement the *Calculate-Test-Adjust* strategy well compared to almost all other teams in the *Competition*.
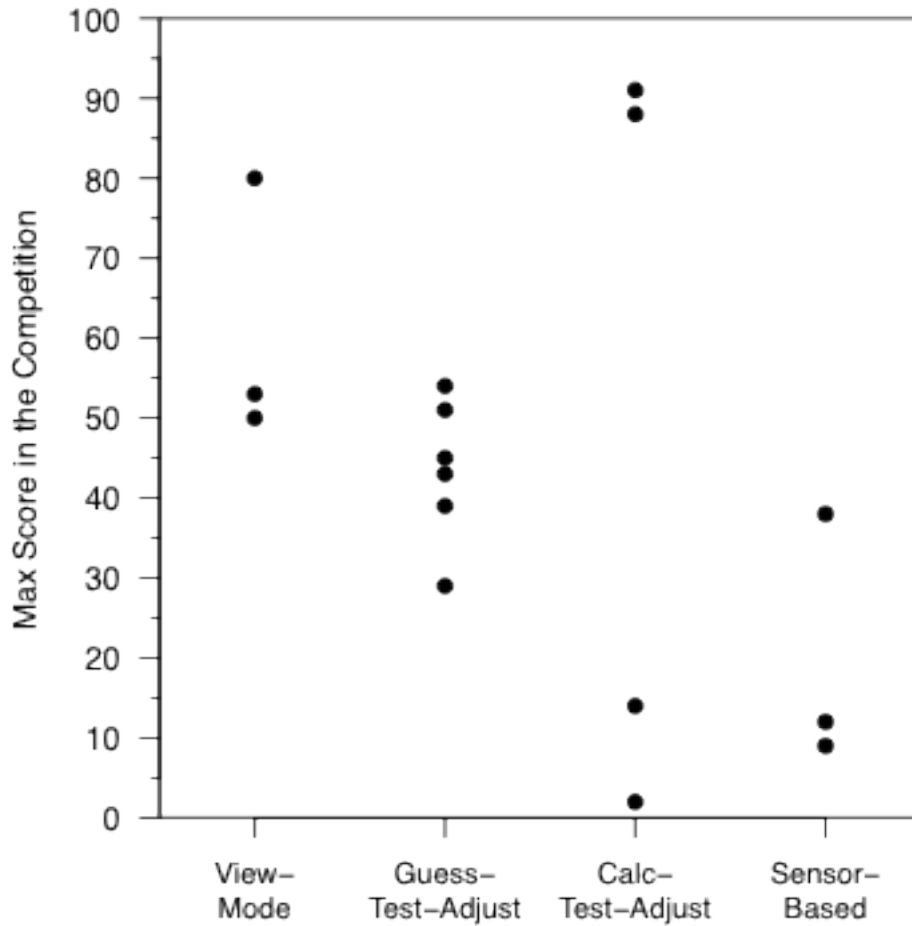
Figure 16. Scatter plot of scores in the *Competition* based on strategy used

This bi-modal distribution for the *Calculate-Test-Adjust* strategy suggests that for this strategy in particular, it may be important to assess how the strategy was used. I hypothesize that when the *Calculate-Test-Adjust* strategy is implemented well, it is just as quick and just as reliable as the *View-Mode* strategy, if not even better. Done without a full understanding,

however, the calculations could involve considerable time and cognitive resources that distract from committing those resources to other parts of the design solution.

In sum, a plausible explanation is that teams who are fluent with mathematics can use math-based measurements and predictions to their advantage by determining the correct motor rotation values for different moves relatively quickly. As with the *View-Mode* strategy, this timesaving frees resources for use on building tasks and fine-tuning overall strategy. Teams that are less fluent in mathematics, however, may take longer to perform the math-based calculations, and make more errors, thus taking time away from working on other important parts of the task.

2.5.4   Discussion

To summarize the results, only a few teams used math explicitly in their design solutions. The use of math was found to have a highly variable relationship with design success, with the highest and very low scoring teams in the competition having used math. But this does suggest that there is a possible role for math as a tool for controlling robot movements even in situations that don't explicitly favor doing so. Given that, the challenge for instructional designers may be how to get students to choose math-based strategies even when other non-math-based strategies (e.g., the *View-Mode* strategy) are readily available and work well.

2.6   STUDY 4 – MODEL ELICITING

Studies 1-3 provided the groundwork for a revised learning environment design. Participation in robot competitions did not reliably lead to understanding how the robots work more generally as

teams often developed solutions fine-tuned for the particular challenge without finding a need to understand how the general system works. The *Design Based* learning environment was reasonably successful at both engaging students and facilitating use of math in their robot strategies, but was somewhat inefficient as students still spent much of their time-on-task constructing fine-tuned solutions without attending to the more general conceptual ideas. The challenge for a revised learning environment would be to put that conceptual focus at the core of what students did in the unit.

The building of the dance routine in the *Design Based* unit from Study 2, although an engaging part of the task for the students, didn't serve the purpose of getting students to connect proportional reasoning to robot movements. Students instead put a lot of effort into fine-tuning their dance routines (using a lot of guess-and-check). The unit then forced them to go through a lengthy process of measuring their dance routine so they could know when a robot was doing it right. That activity was not something they enjoyed or thought was useful. Furthermore, the measuring of the design specification did get the students some sense of how to program the robot and how to measure its movement, which may be important learning objectives in their own right, but weren't the ones that were intended. It wasn't till the eleventh session that the students finally did their first synchronization attempt. It wasn't till the following session that they started to generate solutions to that synchronization problem. This is possibly too much peripheral activity when the goal of the instructional design is really to get students focused on using the mathematics of proportional reasoning as a tool for understanding robot movements.

I was inspired by research on the design of model-eliciting activities (MEAs), which showed how subtle changes to the framing of problems can have a large effect on the sorts of solutions that get elicited (Lesh, Hoover, Hole, Kelly, & Post, 2000). Lesh et al. articulated

principles for the design of model-eliciting activities (MEAs)—authentic problems carefully chosen such that the situation itself motivates a need to create a general mathematical model (Lesh et al., 2000). MEA design principles suggest that many typical activities with robots (e.g., robot mazes or more complex robot competition boards) may not be ideal for eliciting conceptual models for two reasons. First, although the robot tasks can be complex with many different parts, they don't encourage students to generalize their understanding in a way that would be reusable in other situations. Instead, the activity structure encourages students to develop solutions that are finely tuned to the particular challenge. A second reason is that typical robot activities don't provide incentives for students to explain their ideas to others, and as a result, how well the solution works is valued highly, but how well the solution is understood at a conceptual level is not. This suggests that the domain of robotics may be especially prone to encouraging guess-and-check sorts of behaviors in which students are not likely to attend to and reflect on their underlying situation models. Another implication is that alternative instructional experiences that are more consistent with MEA principles may be better suited to helping students develop and refine their understandings, which in turn may motivate the use of mathematics as a tool for that understanding. This study attempts to put students in an activity structure more consistent with MEAs, and by doing so, seeks to observe students begin to make the transition from strategies that focus solely on getting the immediate challenge correctly solved to strategies that reflect on the underlying conceptual understanding that enables solving more general problems.

I redesigned the RSD unit with the key modification being that the design goal was no longer about designing a dance routine. If the students believe their goal in the activity is to design one fine-tuned set of robot movements (i.e., an interesting dance or even a synchronized dance), then it seems unlikely that the students would be focused on the underlying regularities

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that gave rise to those movements. If instead, students believe that the thing they create is a tool to coordinate the physical features, the program parameters, and the robot movements, then their activity is more likely to be aligned with the sorts of experiences that would help them advance their understanding of these ideas. I represent this subtle shift in goal in Figure 17.
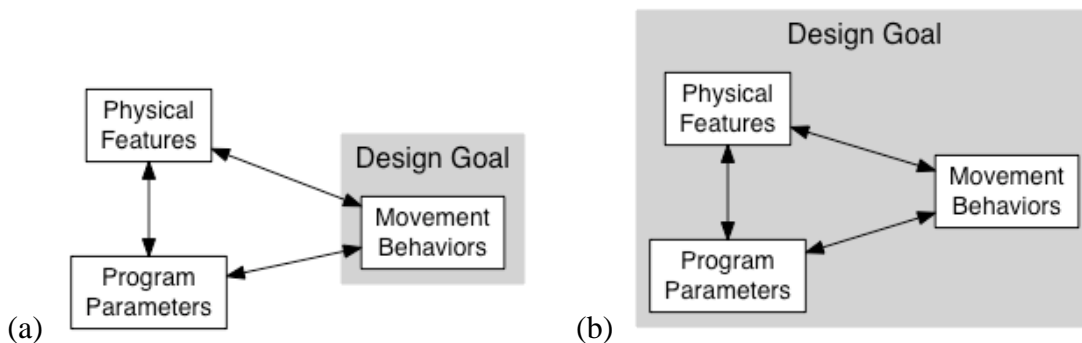
Figure 17. Design goal as (a) a set of movements versus (b) a coordination of features

Two other modifications were made in this revised RSD unit. The first is that based on the analysis if the *Design-Based* unit it seemed that there were opportunities after students had invented their own solutions to provide examples of well-formed solutions that would have helped students clarify and articulate their own thinking. As a result, teacher-provided strategies were a part of the revised design.

A second modification was more a shift in emphasis, in that the posters that students created to share their current synchronization strategies and the discussions held around the sharing of those posters became a more central and valued aspect of this revised learning environment. The posters and the talk became the main source for understanding students' thinking and the ways in which they were connecting math in robots. It was also a forum for

disseminating those ideas to the rest of the class, and a way to indicate to the students that their explanations and justifications for the strategies were at least as important if not more important than just being able to program the robots to dance in sync.

2.6.1 Activity context

For this study, the *Robot Synchronized Dancing* (RSD) instructional unit was revised using a model-eliciting activity framework (Silk, Higashi, Shoop, & Schunn, 2010). As in the *Design Based* version of the unit, this RSD unit facilitates students in programming multiple LEGO robots to dance in sync with each other. To investigate students' use of mathematics, the unit was redesigned as a model-eliciting activity (MEA) in which students invent solutions in a series of express-test-revise cycles (Hamilton, Lesh, Lester, & Brilleslyper, 2008; Lesh et al., 2000). Students work in teams of 2-3 and create a "toolkit" for a robot dance team captain with a team of different-sized robots. The captain needs a synchronization solution for any dance routine. This helps focus the teams on designing a general, adaptable, and explainable solution. All of the materials for this *Model Eliciting* environment are included in Appendix E.

Instead of creating their own dance routine, teams are provided with an example dance routine—set to the "Cupid Shuffle"—that works on one robot—a shared robot accessible to everyone in the class. But the students' job is to create a toolkit or set of strategies for getting other robots to also do the dance the same way. All the other teams have their own robot, and all are of the same design—a robot with different physical characteristics as the shared robot. As a result of sharing the same robot type and the same example dance routine, there is a shared problem that may then be easier to communicate about across teams within the class.

Again, the robots and the dance moves in the example dance routine are carefully chosen to make visible key proportional relationships between the robots' physical design, the program parameters, and the magnitude of the robots' movements. Hence, proportional reasoning (Lamon, 2007) is still a key mathematical model that teams are facilitated in using. Teams are presented with the entire problem up front, including a full measurement specification of the dance routine, but the activities are structured into sub-problems. It was hypothesized that this unit would encourage mathematical modeling more immediately and to a greater extent than competition activities, and that this in turn would lead to increases in students' understanding of how the robots work. It was also hypothesized that the aspect of synchronization would replace the aspect of creating a dance routine as the central design goal and that that synchronization goal itself would be engaging to students.

2.6.2   Method

2.6.2.1   Participants

A total of 29 sixth- through ninth-grade students participated from 3 sections of an elective robotics course in an urban grade 6-12 school. The students self-selected into one of three sections, two of which were the *Model Eliciting* condition ($n = 21$) where students participated in the RSD instructional unit, and the other one was the *Competition* condition ($n = 7$) where students worked as a team to prepare and compete in a robot competition—a state FLL competition. The students in the *Model Eliciting* condition worked in groups of 2 or 3 students per group, which resulted in a total of 11 groups. As in Study 3, the students in the *Competition* condition worked within their team in a variety of configurations of groups of students, but all

the analyses for this study that included the *Competition* condition were completed at the individual student level or at the team (section) level.

The study took place within the school context of a STEM-focused magnet program within an urban school district. Because the program was open to all students throughout the district, the participants were from a wide geographic range of neighborhoods from the city. This was their elective course for the program, so was not a formal part of their course of study.

There were 44 total students in the 3 sections of the elective course, but 15 students (12 in the *Model Eliciting* condition and 3 in the *Competition* condition) were excluded from the analyses because they did not complete the pre- and post-assessments.

2.6.2.2 Data sources

Problem solving assessment

The *Problem Solving Assessment* was identical to the one used in the previous study (Study 3). Based on the sample in this study, the assessment was adequately reliable for both groups (Cronbach's $\alpha = 0.78$ for the *Model Eliciting* group; Cronbach's $\alpha = 0.61$ for the *Competition* group).

Attitudes survey

The *Attitudes Survey* was identical to the one used in the previous studies. Based on the sample in this study, the survey was adequately reliable for both groups at the overall level (Cronbach's $\alpha = 0.82$ for the *Model Eliciting* group; Cronbach's $\alpha = 0.48$ for the *Competition* group) and on the three subscales: *robotics interest* (Cronbach's $\alpha = 0.73$ for the *Model Eliciting*

group; Cronbach's $\alpha = 0.78$ for the *Competition* group), *math interest* (Cronbach's $\alpha = 0.78$ for the *Model Eliciting* group; Cronbach's $\alpha = 0.53$ for the *Competition* group), and *math value for robotics* (Cronbach's $\alpha = 0.71$ for the *Model Eliciting* group; Cronbach's $\alpha = 0.56$ for the *Competition* group).

Observations and student work

The *Model Eliciting* group activities were video recorded and the work they produced was collected, including worksheets and posters describing their evolving RSD toolkits. A final poster created by the *Competition* team was also collected. The poster describes their solution for the competition task that they used to present to judges at the competition.

2.6.2.3 Study design

Performance on the *Problem Solving Assessment* and responses on the *Attitudes Survey* were used as the dependent measures of disciplinary learning and disciplinary engagement respectively and were contrasted between the learning environments. The other data sources were used to identify the nature of the connections that students made between math and robots, and to identify the features of each learning environment that influenced those connections.

2.6.2.4 Procedure

Preparing for the competition requires more time, so the *Competition* condition participated in 32 hours of robot activities, whereas the *Model Eliciting* condition participated in 8 hours. The first author taught the *Model Eliciting* sections and the school's engineering instructor taught the *Competition* section. Both groups worked with the same LEGO robots; moreover, a large part of

the competition required getting their robot to move specific distances and angles, which was the primary focus of the *Model Eliciting* group activities. Both groups were given the pre-assessments on the same day. Students were given 25 minutes for the *Problem Solving Assessment* and 5 minutes for the *Attitudes Survey*, one immediately following the other. They completed both surveys individually. On the *Problem Solving Assessment* they were instructed to give an answer for every question, to show their work, and were permitted to use a calculator. The post-assessments were administered the day after the conclusion of each condition's respective activities. For the *Model Eliciting* condition that occurred approximately one month from when the pre-assessments were administered. For the *Competition* condition that occurred approximately two months from when the pre-assessments were administered.

2.6.3   Results

2.6.3.1   Problem solving assessment

Descriptive data from the *Problem Solving Assessment* are reported in Table 8. These data were analyzed using a repeated measures ANOVA with proportion correct on the problem solving assessment as the dependent measure, time (pre, post) as a within-subjects factors and condition (*Model Eliciting* or *Competition*) as a between-subjects factor. In this analysis there were no significant main effects or interactions between time and learning environment condition. However, adjusting for multiple comparisons using a Bonferroni correction, there was a significant mean difference from pre to post within the *Model Eliciting* condition ($M = 0.10$, 95% CI [0.01, 0.20]), and the mean difference from pre to post was not significantly different than zero in the *Competition* condition ($M = 0.02$, 95% CI [−0.15, 0.18]). This indicates that

participation in the *Model Eliciting* RSD unit did have a positive effect on students' overall problem solving, but the effect was not reliably different from participating in the *Competition* environment.

Table 8. *Model Eliciting* problem solving outcomes results

Measure	Model Eliciting (n = 21)				Competition (n = 7)			
	Pre	Post			Pre	Post		
	M (*SD*)	*M* (*SD*)	*r*	*d*	*M* (*SD*)	*M* (*SD*)	*r*	*d*
Problem Solving	0.5 (0.3)	0.6 (0.3)	0.7	0.4*	0.5 (0.2)	0.5 (0.3)	0.8	0.1

$^+ p < .10.$ * $p < .05.$ ** $p < .01.$ *** $p < .001.$

2.6.3.2 Attitudes survey

Descriptive data from the *Attitudes Survey* administered pre and post are reported in Table 9. These data were analyzed using a multivariate repeated measures ANOVA with the mean rating on each of the attitude scales and subscales as the dependent measure, time (pre, post) as a within-subjects factors and condition (*Model Eliciting* or *Competition*) as a between-subjects factor. At the overall level there was a significant main effect of time, $F(3,24) = 5.09$, $p < 0.01$, $\eta^2 = 0.39$, but the effect of condition, $F(3,24) = 1.46$, $p = 0.25$, and the interaction were not significant, $F(3,24) = 0.47$, $p = 0.70$. The univariate tests suggest that the overall significant effect was due to a significant effect of time for the *robotics interest*, $F(1,26) = 7.28$, $p = 0.01$, $\eta^2 = 0.22$, and *math value for robotics*, $F(1,26) = 5.51$, $p = 0.03$, $\eta^2 = 0.18$, subscales, but not for

the overall attitudes scale, $F(1,26) = 0.01$, $p = 0.94$, or the *math interest* subscale, $F(1,26) = 0.06$, $p = 0.82$. Further, the only mean differences from pre to post that were significantly different from zero were a negative change within the *Model Eliciting* condition for the *robotics interest* subscale ($M = -0.36$, 95% CI [−0.64, −0.07]) and a positive change also within the *Model Eliciting* condition but for the *math value for robotics* subscale ($M = 0.36$, 95% CI [0.06, 0.65]). This indicates that participation in the *Model Eliciting* RSD unit had a negative effect on students' interest in robotics, but had a positive effect on their sense of the value of math for robotics. For the *Competition* condition, there were no significant differences from pre to post on their overall attitudes or on any of the individual subscales, which suggests that there is not enough evidence to determine whether participating in that environment impacts engagement.

Table 9. *Model Eliciting* attitudes outcomes results

Measure	Model Eliciting (*n* = 21)				Competition (*n* = 7)			
	Pre	Post			Pre	Post		
	M (*SD*)	*M* (*SD*)	*r*	*d*	*M* (*SD*)	*M* (*SD*)	*r*	*d*
Overall	0.6 (0.6)	0.6 (0.6)	0.8	0.0	1.0 (0.4)	1.0 (0.4)	0.8	0.0
Robotics Interest	1.1 (0.7)	0.7 (0.6)	0.5	−0.6*	1.5 (0.5)	1.1 (0.9)	0.7	−0.6
Math Interest	0.4 (0.9)	0.3 (0.9)	0.9	−0.1	0.6 (0.7)	0.7 (0.7)	0.9	0.2
Math Value for Robotics	0.4 (0.8)	0.7 (0.7)	0.6	0.5*	0.9 (0.8)	1.2 (0.5)	0.7	0.5

$^+ p < .10.$ * $p < .05.$ ** $p < .01.$ *** $p < .001.$

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2.6.3.3 The *Competition* group

The *Competition* team did very well in the competition finishing 11th out of 72 teams. In addition, they did use math explicitly in their solution strategy (Figure 18). Based on the proportion of teams that used math in their solutions in Study 3 (25%), it might be reasonable to conclude that the *Competition* team in this study was exceptional relative to other teams in the competition in terms of their use of math. It was also the case that their coach was aware of the focus of the *RSD* unit and so may have inadvertently encouraged the students to incorporate some of the same math ideas in their competition solution. This sort of cross-contamination across the study conditions did occur at least once as the *Competition* team chose to use the same base robot design that was being used for one of the robot dancers in the RSD unit.
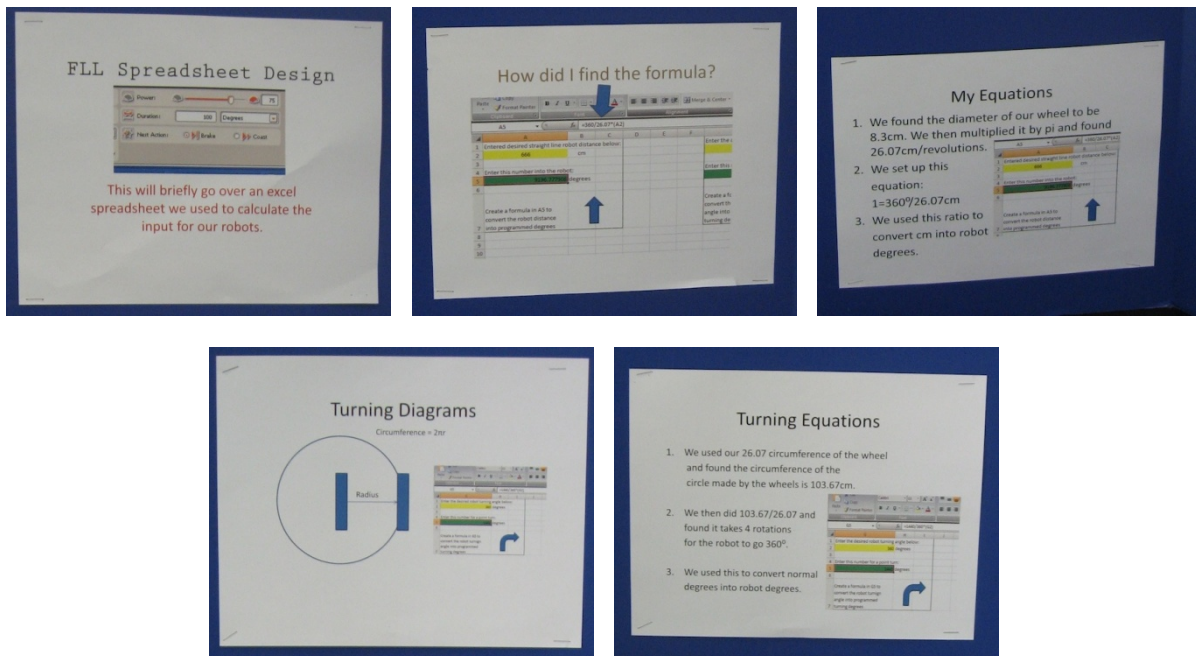
Figure 18. Math-based solution by the *Competition* team in Study 4

2.6.3.4   The *Model Eliciting* group

In addition to the pre-post gains, one measure of the success of the redesign of the unit was the immediacy with which students incorporated proportional reasoning into their activities and their solutions and the sophistication of those solutions in terms of connecting to math. Unlike in the *Design Based* environment in which it took more than 11 sessions for the students to first encounter the synchronization problem, in the *Model Eliciting* environment the students are introduced to the synchronization problem on the very first instructional day (after the pre-test day) and after only a short introduction. Furthermore, although some guess-and-test strategies were observed at various points in the implementation, the vast majority of invented strategies in the *Model Eliciting* condition consisted of complete forms of scalar or functional reasoning. This suggested that indeed the revised learning environment design did a much better job at providing a more conceptually focused activity on the connections between math and robots.

2.6.4   Discussion

The implementations of the RSD unit helped students improve their understanding of the way the robots work. These improvements were the result of providing a context where students' were immediately encouraged to connect math to their thinking about the robots in more explicit terms, a practice less likely to occur when preparing for robot competitions.

One curious finding that is in need of some explanation is the decrease in robotics interest from pre to post in the *Model Eliciting* condition. Similarly to the results from the *Design Based* environment in Study 2, the students in the *Model Eliciting* condition did make significant improvements from pre to post in their sense of the value of math for doing robots. This may

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indicate that the revisions to the task maintained the core aspects that connected the math to the robotics in this situation in the same way as was done in the *Design Based* environment. However, it may have been that removing the part of the activity where the students get to design their own dance routines influenced the engagement level in terms of how the *Model Eliciting* students felt about the fun of doing robots. It may be that they thought of the revised *Model Eliciting* version of the RSD task as being more about "hard work" than "hard fun," even if they could better appreciate the value of math in robotics as a result of that experience.

2.7   GENERAL DISCUSSION

The four studies presented in this part provide a rich description of the landscape of opportunities that are available for students to connect math in robotics that range from formal to informal. Although the quantitative results of the effect of the *Model-Eliciting* environment were not overwhelmingly strong, that environment does a much better job of more immediately aligning students' activity with the type of thinking that was desired. In addition, it accomplished this while overall maintaining students' engagement levels. There are almost certainly other learning environment designs that could accomplish similar results. Nevertheless, the analysis of the four environments suggests that a core principle in the design of learning environments that help students connect math with robots is about knowing how to fine-tune the tasks such that almost immediately the students are focused and working on that aspect of the problem.

Returning to the original theoretical graph of learning and engagement presented in the introduction (Figure 1), I am now in a position to show an empirical version of that graph based

on the results from the four studies. Figure 19 shows engagement and learning as mean normalized gain scores for each group. The normalized gain score was used to control for correlations between pretest and gain scores.
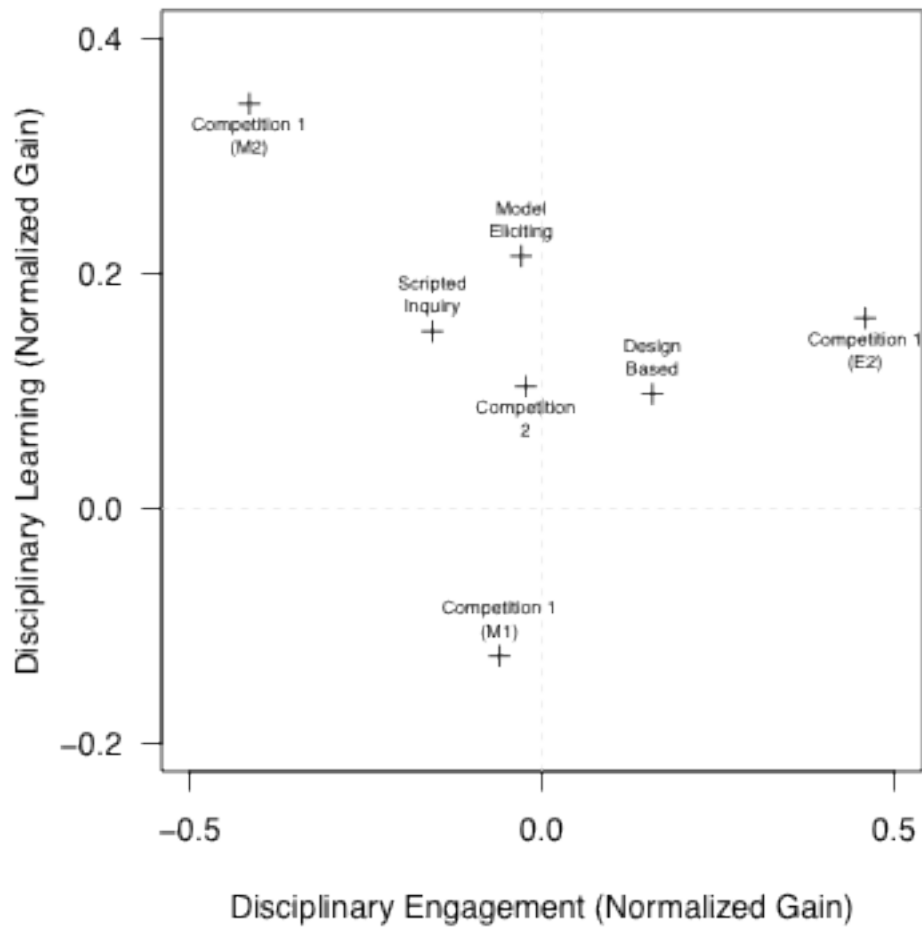
Figure 19. Effects of the environments on learning and engagement

In this graph it becomes apparent that the *Model Eliciting* group, although not ideal, is in a reasonably successful space on the graph. Given that the *Competition* groups may be considered the gold standard for engagement, and that very few of those groups are to the right

of the 0 line on the engagement axis, it may be that a more reasonable target for learning environment designs is to maintain existing levels of engagement, especially as those designs try to encourage students to engage in increasingly more rigorous and challenging forms of activity. In this case, connecting math with robots is a rigorous and challenging activity, and the *Model Eliciting* environment seems to have performed very well according to that standard.

3.0   PART 2 – LEARNING FRAMES FOR MATH IN ROBOTS

The second part of this dissertation project focused on different ways that students connect math with robotics within the *Robot Synchronized Dancing* (RSD) unit. Since the prior studies illustrated how the RSD unit is an effective environment for helping students to make connections between math and robots, it was a useful context for studying differences in students' approaches to doing so. This part of the dissertation included two studies. The first study focused on identifying contrasting approaches observationally from analysis of data collected in a prior study using the RSD unit. The second study follows directly from the first by implementing contrasting instructional implementations of the RSD unit that each encourage the participating students to take on one of the observed approaches.

3.1   OVERVIEW OF STUDIES

The following is a brief summary of the main findings from each study and the connections between them to help guide the reader:

Study 5 – Identifying Contrasting Frames. I conducted a retrospective analysis of the student work produced when implementing the RSD unit in Study 4. In this new study, the goal was to identify more and less productive approaches to the tasks. I engaged in a search for

"ontological innovation" (diSessa & Cobb, 2004)—a search to identify a distinctive way to categorize student approaches such that the category is explanatory of some meaningful aspect of the learning that is happening in this context. Further, the new category should have implications for alternative designs of the environment that would likely be consequential for learning. In this case, I found that students' epistemological framing of the task appeared to be consequential even when students' essentially performed the same numerical operations and procedures. Two contrasting approaches to the task included one that was much more common: using math as a calculational tool for performing numerical calculations on numerical values, for defining a pattern of numerical values, and for reproducing that pattern in a well-specified input-output function. Students that could do this well seemed to do very well in the RSD task. An alternative approach, however, emerged even though it was much less common. Some students used the math not just as a calculational tool, but also as a way to represent their ideas about the way the robots work—the robot's mechanisms of movement. They reasoned with mental images and animations that guided what sorts of quantities in the situation were relevant and what sorts of operations could be performed on those quantities that would mirror what was happening in the situation. This finding is consistent with research in mathematics (Thompson et al., 1994) and science education (Hutchison & Hammer, 2009) that have both identified similar epistemological frames as consequential for learning in those disciplines.

Study 6 – Manipulating the Contrasting Frames. Following up on Study 5, I then tested whether these contrasting epistemological frames were consequential for learning by performing an instructional manipulation between two different implementations of the RSD unit. The results supported the conclusion that the mechanistic approach condition was the only condition to show a significant problem solving improvement from pre to post. Coding of the posters

created by the teams confirmed that the mechanistic group teams did generate more mechanistic strategies primarily by including more pictures of the situation in their poster and physical features of the robot in their strategy. They were also much less likely to include guessing or adjustments as part of their strategy. Perhaps most powerfully, in a competition task that took place after all of the RSD activities had concluded, including the post-assessments, the four mechanistic teams all used the strategies that they had invented in the RSD unit, but only 1 of the 4 calculational teams did so. Analysis of the whole-class discussions revealed that the calculational group did engage in high-level mathematics as part of the RSD unit, so the differences were not due to general disengagement from the task or lower levels of mathematics ability. Rather, the mechanistic teams appeared to activate a greater range of conceptual explanatory resources available for thinking about the problem. Comparison of the quantitative results with the environments from Part 1 suggested that the calculational group was equivalent to the *Model Eliciting* group from Study 4, and that the mechanistic group improved on those results, demonstrating higher normalized gains than any other group in the prior studies while still sustaining engagement levels.

3.2   BACKGROUND

The work presented in this study was influenced by prior theoretical and empirical work both in mathematics and science education research. One major influence was the contrast between calculational and conceptual orientations to teaching mathematics (Thompson et al., 1994). In this framework and as illustrated in contrasting case studies, teachers with calculational

orientations tend to speak exclusively in the language of numbers and numerical operations. They also have a predisposition to frame problem solving as entirely about producing a numerical answer. In contrast, teachers with conceptual orientations are more focused on articulating ideas and ways of thinking. They reason with images and have expectations that problem solving will be intellectually engaging. Thompson et al. (1994) also suggest that even though the conceptual orientation is by far the more productive for teaching and learning, that it is incredibly difficult to acquire and once acquired it is difficult to maintain. Although not explicitly set in a math education context, similar contrasts in orientations have been observed in the context of the cross-discipline studies reported here. I will argue that that these contrasting orientations from the mathematics education literature are likely to be consequential for learning in this context based on prior research on using math as tool for thinking about a physical situation from cognitive psychology research and on research investigating the effect of mechanistic reasoning in science education research.

As reported earlier in this paper, Schwartz et al. (2005) demonstrated the positive effect of math in physical systems in which multiple features must be attended to and coordinated to predict an outcome. Tools that organize thinking for learners can improve their understanding. In the Schwartz et al. (2005) study, they showed how mathematics can be such a tool by manipulating conditions that would make it more or less likely for students to use math in their strategies for solving the balance scale task in which they have to determine which side of the balance scale will fall. To explain further, in one study they manipulated whether the balance scale was displayed using a computer in its default format with easy-to-measure, discrete quantities for both weight (number of weights on a peg) and distance (number of pegs from the center) versus in a modified format with difficult-to-measure, continuous quantities for weight

(amount of liquid in an unmarked beaker) and distance (distance from the center without any marked pegs). Ten-year-olds in the easy-to-measure condition exhibited results similar to their age-group norms in which they attended to both weight and distance across different problems, but had difficulty choosing which side would balance when both were different and so they were required to attend to those quantities simultaneously. In contrast, ten-year-olds in the hard-to-measure condition performed closer to typical five-year-olds, most often paying attention exclusively to weight in their problem solving and not attending to distance at all. Schwartz et al. reasoned that the students in the hard-to-measure condition likely were disadvantaged because they were not able to represent both quantities with the same ontology (numbers) and so that made it very difficult to attend to both quantities and choose between them. In this sense, numbers provide a way to organize the data available in a situation in a common form so that quantities can be compared and potentially combined more easily than in their basic perceptual forms.

In a second study, Schwartz et al. (2005) gave two groups of eleven-year-olds the easy-to-measure version of the balance scale task, but in one group students were asked to justify their answers with the prompt, "Explain your reasoning," and in the other condition they were prompted to "Show your math." In this case, the "Show your math" condition far exceeded their age-group norms and solved the problem at a level similar to and possibly even exceeding typical adults, in which they attended to both weight and distance simultaneously. Despite significant developmental gains simply by encouraging the participants to use math, the majority of the eleven-year-old participants in that study did not reach the highest level of reasoning in which they recognized that the most appropriate operation to combine weight and distance was using a product. Schwartz et al. suggested that the mathematics was beneficial in helping the

students to consider possible alternative structures for coordinating features (i.e., mathematical operators), but did not provide a basis for choosing between those alternatives. Essentially the students were left to test out each alternative quantitative operator empirically and so it turned into a game of chance whether or not the students found the correct one. Building on this work, it is possible that there are conceptual resources that students could draw on that would help them continue to use math as a tool for organizing their thinking, but then also help supplement that tool by focusing on testing the quantitative operators that seem most likely to work in the situation. The conceptual orientation described above, and more appropriately, a mechanistic orientation as described in the science education literature, may be the sort of supplement to math that is needed to provide a basis for choosing between alternative mathematical structures.

Orientations that allow for using mathematics in more principled ways may provide an additional benefit of this type. In a flood prediction task, Kaplan and Black (2003) provided middle-school-aged students cues about the mechanisms by which each feature may impact water levels. The mechanistic cues caused students to engage in more mental animations of the system, which led to more focused investigations of causal effects of individual features and better predictive accuracy during those investigations. Using mathematics specifically to represent mechanisms, a combination of the superior conditions from Kaplan and Black (2003) and Schwartz et al. (2005), may facilitate reasoning beyond what was observed by either in isolation. Using mathematics as an organizational tool together with mechanistic reasoning to focus efforts on the most likely organizations for a given situation, may propel development further as students can rule out many implausible feature effects and interactions using internal animations and focus on testing only plausible ones.

Returning back to the specific situation in the Schwartz et al. (2005) study, the research on mechanistic approaches Kaplan & Black (2003) suggests that if two groups were both prompted to use math in easy-to-measure situations like they were in the second Schwartz et al. (2005) study, but one group was also facilitated in using mechanistic reasoning while the other group was not, then the mechanistic group may be able to do a more focused search of the potential mathematical operators and arrive at the correct mathematical relationship sooner. They would have access to additional cognitive resources that would help them to determine which operators are possible and eliminate those that wouldn't make sense within the physical situation. In contrast, the non-mechanistic group would be more likely to engage in a trial-and-error search of the possible mathematical operators and so would take longer or be less likely to arrive at the correct relationship.

The present study used a physical system context in which students were likely to have intuitions about mechanisms that relate system features—middle school students learning to program simple robot movements. Students have intuitive ideas about how wheel rotations and wheel size relate mechanistically to produce movement distances, but these ideas are rarely fully articulated or connected explicitly to students' solutions. Thus, I was able to investigate different orientations for math use: (1) a *mechanistic* orientation in which math is used as a tool for modeling physical intuitions about the way the system works; versus (2) a *calculational* orientation in which math is used as a tool for describing input-output patterns induced empirically. I used a teaching experiment design in order to investigate the development in reasoning and also the interactions between students as they communicated their ideas.

3.3   STUDY 5 – IDENTIFYING CONTRASTING FRAMES

Qualitative observations of the solution strategies invented by the teams within the *Model Eliciting* condition from Study 4 suggested that there existed meaningful differences in mathematics use between them and that those may have mapped on to the distinctions between mechanistic and calculational orientations to using math. Examining contrasting types of solutions and attempting to characterize the nature of those differences may help identify student behaviors or that may be more productive for learning. This study adopts that approach.

The particular teams to contrast—Team A2 and Team B1—were selected based on the completeness of their data and their comparable group characteristics. The two selected teams were two of only five teams that completed all three posters in the *Model Eliciting* implementation, and the only ones of those five of whom all of the team members completed both the pre and post tests. In addition, both groups contained 1 sixth grade male, 1 seventh grade male, and 1 ninth grade female, so they were roughly comparable in terms of group composition.

3.3.1   Method

3.3.1.1   Participants

Each team in Study 4 was assigned a unique label based on the classroom section in which they participated. The team label was put on each of their posters and other work, so the labels used in the classroom were the same ones used in this study to identify the two selected teams. The alphabetic character in the label corresponds to which team within a section that the students

were on, with teams "A" through "F" in each section, and the numeral "1" or "2" distinguishes between the two classroom sections. The teams were assigned their alphabetic character randomly. Thus, Team B1 and Team A2 were not in the same classroom section.

None of the three students in Team A2 had prior robotics experience. All were in a typical math class for their grade level, with the ninth grade female enrolled in an Algebra 1 class. In Team B1, neither of the two males had prior robotics experience, but the female indicated that she had used robots once at an out-of-school girls science program. She was also enrolled in a more advanced math class for her grade level, having completed Algebra 1 in seventh grade.

Table 10. Contrasting teams' group composition

Contrasting Teams	Team A2	Team B1

Team Composition	Sixth grade male
	  no robot experience
	  grade-level math
		Sixth grade male
		  no robot experience
		  grade-level math

	Seventh grade male
	  no robot experience
	  grade-level math
		Seventh grade male
		  no robot experience
		  grade-level math

	Ninth grade female
	  no robot experience
	  grade-level math
	  (Algebra 1)
		Ninth grade female
		  some robot experience (out-of-school girls science program)
		  advanced math (Algebra 1 in seventh grade)

3.3.1.2 Data sources

Students worked in teams on creating their RSD solution designs. After each subsystem, the teams created a poster representing their solution design to share with the rest of the teams in a whole-class discussion. They did this at multiple times during the RSD unit activities. These posters were seen as representative of the team's understanding within the RSD situation at that point in time and so were evaluated for the level of quality that they exhibited.

3.3.1.3 Study design

A contrasting case of two student teams—A2 and B1—who generated very different solutions to the RSD task was analyzed. Their solutions were chosen because they both had complete data and because the composition of the students within each team was similar. The analysis was focused on characterizing the nature of the connections between math and robots made by each team and how they differed from each other.

3.3.2 Results

These two teams initially approached the RSD task very differently. Following the structure of the instructional activities, students were asked to focus first just on getting the different robots to move the same distance (ignoring timing and turning). Each team was given a robot to inspect, but rather than permitting them to run the robots independently from the start, all the teams were asked to modify the motor rotations for that robot an initial time and to test their initial try as a whole class. Team A2 used a guess-and-check strategy from the beginning in which they reasoned that their robot was going too fast and so needed to lower its motor rotations. Team B1

instead reasoned, "Bigger wheels go farther because one rotation is larger." They then generated their own strategy for scaling down the motor rotations for the new robot based on the ratio of the wheel circumferences between the two robots. Although Team A2 generated a reasonable test with their guess-and-check strategy, in contrast to Team B1, they don't make mention of or try to incorporate any of the robot's physical features that may have been responsible for the different movement distances.

All teams were then given time to test further their own ideas and to formalize their best synchronizing distance strategy on a poster. In this first cycle of the unit, both teams generated working strategies based on relative scaling (Figure 20). Team A2 adopted a scale factor strategy proposed by another team based on the ratio of the distances the robots moved with the same motor rotations. They recognized that this strategy worked and chose to adopt it. They then created a poster with this strategy, implementing it correctly, but without mention of how the numerical values correspond to aspects of the physical situation (Figure 20a). Team B1 created a poster describing their initial wheel size scaling strategy (Figure 20b). The two strategies are similar in that they are both based on relative scaling between the robots, but one is based on the ratio of the distances when the robots' motor rotations are equal and the other is based on the ratio of the wheel circumferences. Both will work, but Team B1's is based on a physical aspect of the robot (wheel size) that is responsible for the distance the robot moves, effectively incorporating an additional layer of connection to the situation not present in Team A2's strategy.
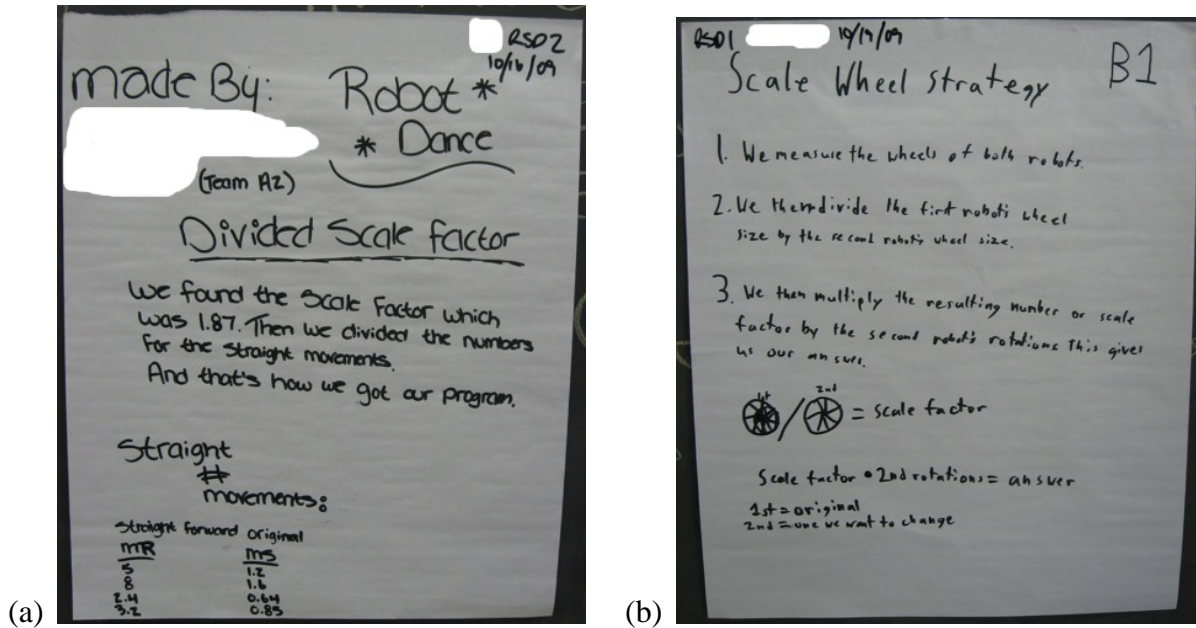
Figure 20. (a) Team A2's and (b) Team B1's first synchronizing distance strategies

After working as a team for a little while, they were given a teacher-provided strategy that described a more formal and more fully explained version of the scaling down strategy based on the relative distance measurements—Team A2's adopted strategy. This strategy was created prior to the implementation based on it being a common strategy used in pilot testing. It was used to highlight the "relative" aspect of the solution as a multiplicative comparison between two robots (Lamon, 1993), and to provide an example of a high-quality explanation. Team A2 recognized it is a more elaborated version of their own strategy without further explanation or critique. Team B1 instead critiqued the teacher-provided strategy as one that "does work" but is not as preferable as a strategy based on wheel size. Team B1 recognized the necessary relative multiplicative aspect of the solution, but applied its own additional criteria for choosing a solution, placing greater value on solutions that incorporate and make explicit physical aspects of the robot that seem to matter.

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After presenting their first posters to the whole class, the teams were asked to revise their strategies to make them more general and easier to explain to others. They were free to modify or change their strategy and to incorporate ideas from other teams. They were also challenged to create a strategy that wasn't "relative" to another robot, but rather was based on just one robot in order to encourage students to consider functional rather than only scalar mathematical models (Tourniaire & Pulos, 1985). Both teams engaged in unitizing—another key component of proportional reasoning (Lamon, 1993)—and generated a unit rate strategy, which is based on figuring out how far the robot moves forward in one motor rotation and then dividing that value into the total distance to figure out the total number of motor rotations needed (Figure 21). However, Team B1 recognized that this unit rate corresponds to the wheel circumference, but Team A2, if they did recognize this connection, did not make it explicit. In addition to not referencing wheels in their verbal explanation of their strategy in their poster (Figure 21a), their picture of the distance the robot moves forward in one rotation also fails to recognize this constraint as the size of wheels drawn on the robot are not even close to what would be needed to traverse that entire distance in one motor rotations. As they did with the first strategy, Team A2 appeared to be content connecting motor rotations and distance traveled directly, using empirical results to test their ideas, and not incorporating physical features of the robot that connect the two.
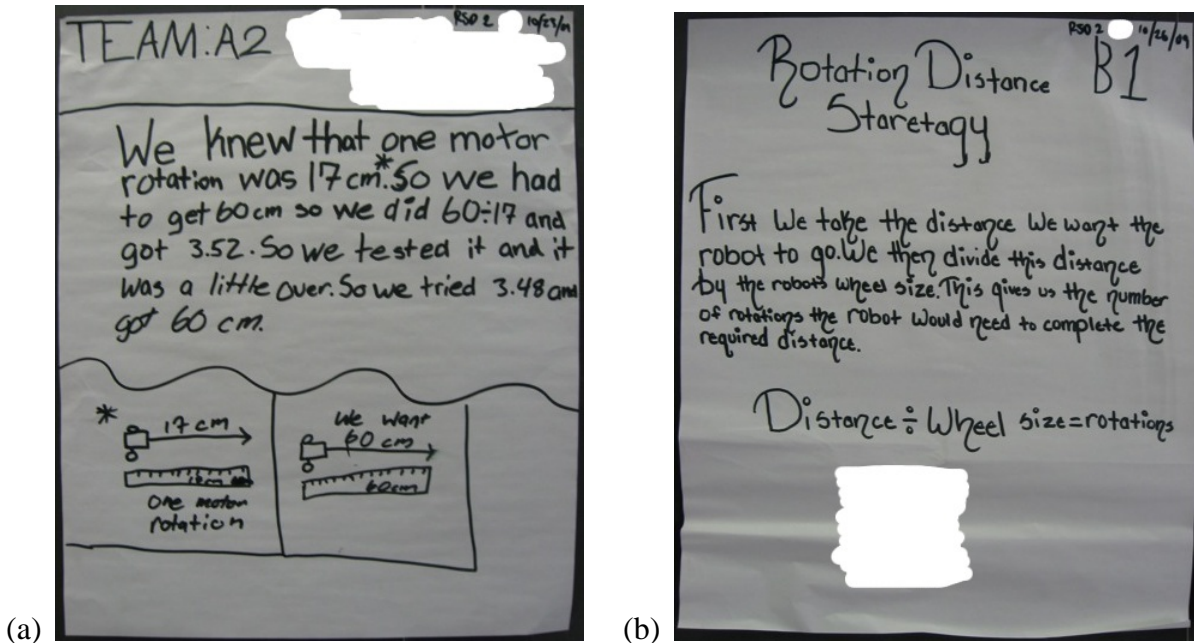
Figure 21. (a) Team A2's and (b) Team B1's revised synchronizing distance strategies

Over subsequent lessons, the teams generated solution strategies for getting different robots to not only move the same straight distance, but also to complete those moves in the same amount of time, and then to synchronize their turning movements as well. Turning is a more complex situation because the amount the robot turns as a function of the motor rotations is based on not only the robot's wheel size, but also on the width of the robot. It is more difficult for students to recognize this property and then to incorporate it in their strategies. Both teams generated final toolkits (Figure 22), but in response to this added complexity, Team A2 reverted back to a guess-and-check strategy. Team B1, on the other hand, makes a reasonable attempt to extend their scaling idea to the turn movements, although they do not successfully identify the robot's width as the key physical parameter that needs to be represented and incorporated.
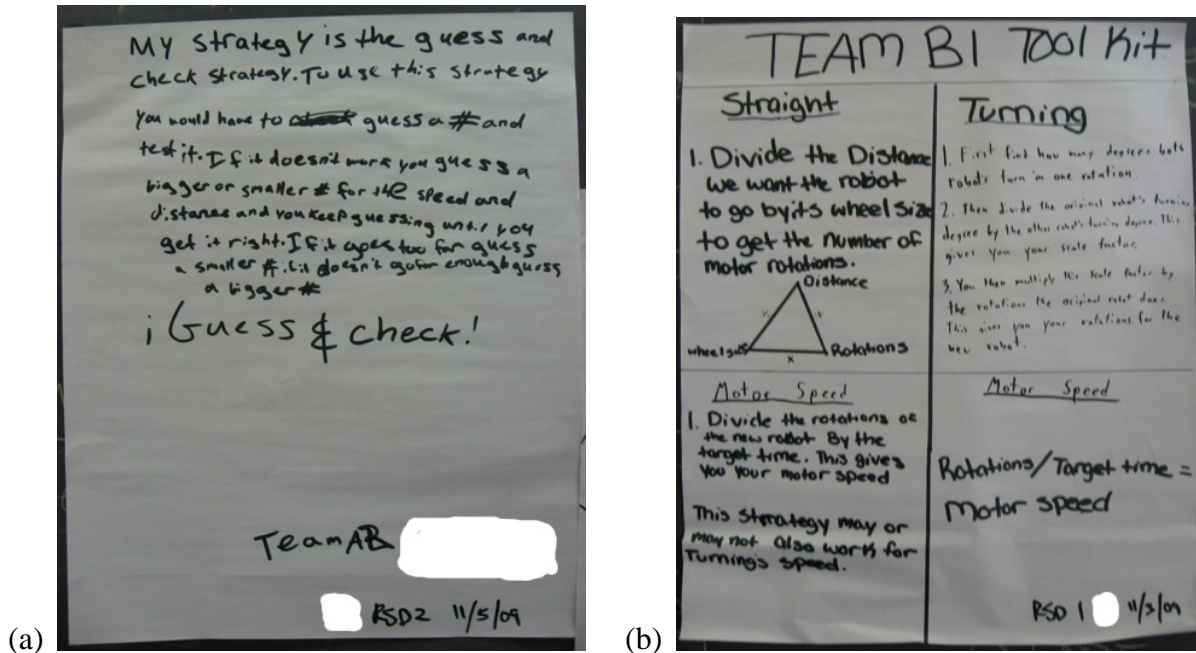
Figure 22. (a) Team A2's and (b) Team B1's final synchronizing toolkits

3.3.3   Discussion

Table 11 summarizes the observed differences between the contrasting teams. Because Team B1

included a student with prior robot experience and an advanced mathematics background, it may

be that access to advanced cognitive resources (e.g., ratios and proportions) explains some of the

differences between the teams. On the other hand, Team A2 was able to use relative thinking and

unit rates (Lamon, 1993) in their intermediate solutions, suggesting that their understanding of

the relevant mathematics was not a limiting factor. Instead, it seems likely that despite of having

access to those mathematical resources, they felt other numerical strategies, including guess-and-

check, were more appropriate for the task.

Table 11. Summarized differences between contrasting teams' approaches in the RSD unit

Contrasting Teams	Team A2 – Calculational	Team B1 – Mechanistic
Initial Ideas for Synchronizing Distance	Used a guess-and-check strategy	Used ratio of wheel sizes (circumference) to scale down motor rotations
	"Because if we picked anything littler than that we though [*sic*] Madonna would go to [*sic*] slow" [3 motor rotations]	"Bigger wheels go farther because one rotation is larger"
First Synchronizing Distance Strategy	Adopts a scale factor strategy based on ratio of distances, but doesn't incorporate any robot physical parameters, or references to the physical situation.	Formalizes their initial "Scale Wheel" strategy with wheel size as the basis
Explaining a Teacher Case *The case uses the ratio of distances w/ same motor rotations to scale motor rotations.*	Recognizes this as a more formal version of their strategy, but without explanation or critique	"This does work but I would rather use the wheel size because distance doesn't apply in turns and can be affected by outside factors."
Revised Synchronizing Distance Strategy	Develops a new strategy with the distance in one rotation as the basis, but without connecting that rate to the wheel size	Continues to use wheel size as the basis
	Does adjustment (fine-tuning) beyond the initial calculation	Is less concerned with getting the values exactly correct
Final Toolkit	Revert to a guess-and-check method without mention of physical robot parameters	Able to extend scale factor reasoning to turns, although doesn't incorporate the additional relevant physical robot parameters (robot width)

An alternative explanation is that the key distinction between the teams was less about cognitive resources and more about epistemological resources—their views about the nature of knowledge and learning that is appropriate for the particular tasks (Louca, Elby, Hammer, & Kagey, 2004). Based on the types of solutions that they generated and their critiques of other strategies, it seems likely that Team B1 held a view of the task as being about trying to represent in their toolkits their ideas of how the robot works. This view is reflected in their use of wheel size in their explanation for the movements, and their defense of wheel size as being a better explanation than considering distance alone. It is for these reasons that Team B1 can be labeled as a *mechanistic* group (Russ, Coffey, Hammer, & Hutchison, 2008). Students with a mechanistic orientation focus on identifying causal mechanisms that underlie natural phenomena and use those mechanisms to focus and constrain the ideas that they consider.

In contrast, Team A2 never feels the need to provide explanations that describe how motor rotations produces distance. They were concerned primarily with getting the particular robots to be synchronized in a precise way as evidenced by their continual use of guess-and-check, and also by their need to fine-tune their obtained values even after applying a math-based strategy (see Figure 21a). Although this view seems consistent with the *textbook correctness* view of tasks from the science education literature (Russ et al., 2008), it may be more appropriately considered as a *calculational* orientation (Thompson et al., 1994) in this environment in which students are connecting math within a physical situation. Students (and teachers) with a calculational orientation have a tendency to focus almost exclusively on the language of numbers and numerical operations without connecting how an understanding of the situation might give rise to those calculations. For these reasons, Team B1 can be labeled as a *calculational* group.

Although there is evidence to support the epistemological labels chosen for these two contrasting groups, it is not possible to fully know based on these data what explains their differences, and even if it were possible, they are only two cases. Other unexamined factors may be relevant. A follow-up study was then designed to encourage two different instructional groups to take on these contrasting epistemological orientations and examine the effect on understanding that results.

3.4   STUDY 6 – MANIPULATING THE CONTRASTING FRAMES

In order to test the hypothesis that a student's epistemological orientation to the RSD task mattered as to what they learned, a second design experiment was conducted. This time, the tasks for the two instructional sections were manipulated so as to encourage one group to take on a mechanistic orientation and the other to take on a calculational orientation.

I adopt a resources view that epistemologies are not individual and stable, but rather that they are context-sensitive and malleable (Louca et al., 2004) and so can be activated within individuals using appropriate instructional moves. This position does not deny that certain individuals may have more of a propensity toward particular epistemological views in more contexts than other individuals.

In general, I hypothesize that differences in learning outcomes can be explained not only by cognitive resources (e.g., prior knowledge, misconceptions, productive conceptual resources, mathematical skills, etc.), but also by epistemological resources (i.e., what one sees as the purpose of an activity in terms of how knowledge is best generated and thought of). More

specifically, I expect that the students who adopt a mechanistic orientation will make greater gains in their understanding. I test this hypothesis through a second design experiment.

3.4.1 Method

3.4.1.1 Participants

A total of 18 fifth- through seventh-grade students participated in 2 sections of the RSD instructional unit. The sections were assigned randomly to conditions—*Mechanistic* condition ($n = 10$) or *Calculational* condition ($n = 8$). Students chose their section based on convenience, but were not informed of the differences between sections. The students worked in groups of 2 or 3 students per group, which resulted in a total of 4 groups per section or 8 total groups.

The students for this study were all recruited from the elementary and middle school campuses of an independent K-12 school. The head of each school campus sent a notice to all parents of students graduating fifth through seventh grade that advertised the research study. The study was conducted in the form of a weeklong educational summer camp, held the week after the school year had concluded.

All of the participating students completed the pre- and post-assessments, so none were excluded from the analyses.

3.4.1.2 Data sources

Problem solving assessment

The *Problem Solving Assessment* was identical to the one used in the previous two studies (Study 3 and Study 4). Based on the sample in this study, the assessment was adequately reliable for

both groups (Cronbach's α = 0.70 for the *Calculational* group; Cronbach's α = 0.82 for the *Mechanistic* group).

Attitudes survey

The *Attitudes Survey* was identical to the one used in the previous studies. Based on the sample in this study, the survey was adequately reliable for both groups at the overall level (Cronbach's α = 0.91 for the *Calculational* group; Cronbach's α = 0.74 for the *Mechanistic* group) and, with only one exception, on the three subscales as well: *robotics interest* (Cronbach's α = 0.91 for the *Calculational* group), *math interest* (Cronbach's α = 0.96 for the *Calculational* group; Cronbach's α = 0.79 for the *Mechanistic* group), and *math value for robotics* (Cronbach's α = 0.70 for the *Calculational* group; Cronbach's α = 0.75 for the *Mechanistic* group). The one exception was that for the *Mechanistic* group the *robotics interest* subscale (Cronbach's α = −0.41) was not adequately reliable, and so inferential statistics were not conducted for that sample on that subscale.

Student work

Data sources included video records of the RSD tasks, posters of teams' invented strategies and other written work, and team post reflection interviews.

In addition, a simplified version of the May Madness competition task (Study 3) was setup and implemented after all of the RSD instructional and assessment tasks were completed. The robot was already built for the students using larger wheels that they had not used in the prior RSD instructional tasks. It had a claw for retrieving toilet paper tubes full of ping-pong balls (Figure 23). In addition, the measurements for the specification of the board were given in

units of inches rather than centimeters as had been true in the RSD tasks up that point. With this setup, students could focus only on the programming aspects, so it was an opportunity to assess how they would solve robot movement tasks after instruction in the RSD unit. By sequencing the task after the final post-tests and introducing it as a "fun" task the students were not biased toward choosing one of their invented strategies from the week.
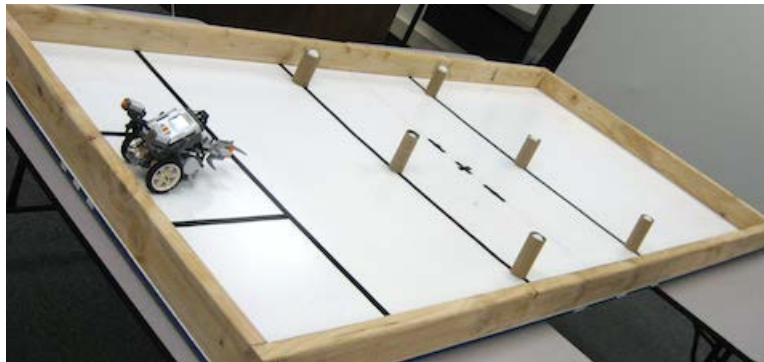
Figure 23. Simplified competition transfer task

3.4.1.3   Study design

Performance on the *Problem Solving Assessment* and responses on the *Attitudes Survey* were used as the dependent measures of disciplinary learning and disciplinary engagement respectively and were contrasted between the manipulated implementations of the RSD unit. The other data sources were used to identify the nature of the connections that students made between math and robots, and to identify the features of each learning environment that influenced those connections. The competition task was used to assess whether students would transfer the strategies they learned in the RSD unit to a less formal task, and whether that would vary by condition.

3.4.1.4 Procedure

Each condition met five consecutive days, two and a half hours per day at a university research building. One group met in the morning, the other in the afternoon. The sections were assigned randomly to conditions. The morning section was assigned to be the *Mechanistic* condition and the afternoon section was assigned to be the *Calculational* condition.

Participating students completed the problem solving and attitudes surveys on the first session and then on the last session of the program. Students were given 25 minutes for the *Problem Solving Assessment* and 5 minutes for the *Attitudes Survey*, one immediately following the other. They completed both surveys individually. On the *Problem Solving Assessment* they were instructed to give an answer for every question, to show their work, and were permitted to use a calculator.

The author was the instructor for both sections. The unit was implemented similarly between sections, except for three distinctions intended to activate the contrasting mathematics orientations (summarized in Table 12). The first distinction was in the design task setup. Each cycle was introduced to the students as a design task, with the *Mechanistic* group asked to represent their intuitions about how the robots work and the *Calculational* group asked to generate steps for getting desired outcome values from input values. The second distinction was in the teacher-provided cases. After inventing their own strategies, students analyzed example strategies that illustrated key understandings, with cases given to the *Mechanistic* group focused on identifying and incorporating key intermediate physical quantities and cases given to the *Calculational* group focused on identifying empirical patterns and numerical operations that generate the patterns. Finally, the third distinction was in the instructional support provided. The questions used by the instructor while students invented their strategies were focused on

connecting quantities and operations to the physical situation in the *Mechanistic* group and

focused on correctness of calculations in the *Calculational* group. All of the materials for the

Mechanistic version of the RSD unit are included in Appendix F.

Table 12. Instructional differences between *Calculational* and *Mechanistic* groups

Instructional Manipulation	Calculational	Mechanistic
Design Task Setup *How each task is introduced to the student teams.*	Focused on input-output transformations "Think of how to transform the motor rotations value into the desired robot distance. Create a strategy for your toolkit that is clear about each of those steps."	Focused on representing intuitions "Think of how motor rotations causes the robot to move forward a specific distance. Create a strategy for your toolkit that captures your ideas about how that works."
Teacher-Provided Cases *Example strategies given to students after they have invented their own strategies.*	Focused on identifying empirical patterns e.g., Scale factor strategy based on the ratio of the distances when using the same motor rotations.	Focused on identifying intermediate physical quantities e.g., Scale factor strategy based on the ratio of the wheel sizes.
Instructional Support *Questions instructors use to assess and advance students when they are inventing their own strategies.*	Focused on correctness of calculations "What are the steps you took to get this value?"	Focused on connecting quantities and operations to the physical situation "What does this value/operation correspond to on the robot?"

Table 13. *Calculational* and *Mechanistic* groups problem solving outcomes results

Measure	Calculational $(n = 7)$				Mechanistic $(n = 9)$			
	Pre M (SD)	Post M (SD)	r	d	Pre M (SD)	Post M (SD)	r	d
Problem Solving	0.5 (0.2)	0.6 (0.2)	0.7	0.5	0.5 (0.3)	0.7 (0.2)	0.7	0.9**

$^{+} p < .10. * p < .05. ** p < .01. *** p < .001.$

3.4.2 Results

3.4.2.1 Problem solving assessment

Descriptive data from the *Problem Solving Assessment* are reported in Table 13. Similar to Study 4, these data were analyzed using a repeated measures ANOVA with proportion correct on the problem solving assessment as the dependent measure, time (pre, post) as a within-subjects factors and condition (*Calculational* or *Mechanistic*) as a between-subjects factor. At the overall level there was a significant main effect of time, $F(1,16) = 11.05$, $p < 0.01$, $\eta^2 = 0.41$, but the effect of condition, $F(1,16) = 0.29$, $p = 0.60$, and the interaction were not significant, $F(1,16) = 1.71$, $p = 0.21$. Follow-up tests suggest that the effect of time from pre to post was significant within the *Mechanistic* condition, $F(1,16) = 12.07$, $p < 0.01$, $\eta^2 = 0.43$, but not within the *Calculational condition*, $F(1,16) = 1.83$, $p = 0.20$. This indicates that there was not enough evidence to conclude that participation in the *Calculational* version of the RSD unit was beneficial to improving students' problem solving, but there was a reliable improvement in the

Mechanistic version suggesting that that version was a particularly effective environment for students to learn about robot movements.

3.4.2.2   Attitudes survey

Descriptive data from the *Attitudes Survey* administered pre and post are reported in Table 14. These data were analyzed using a multivariate repeated measures ANOVA with the mean rating on each of the attitude scales and subscales as the dependent measure, time (pre, post) as a within-subjects factors and condition (*Calculational* or *Mechanistic*) as a between-subjects factor. At the overall level there was a significant main effect of time, $F(3,14) = 6.20$, $p < 0.01$, $\eta^2 = 0.57$, but the effect of condition, $F(3,14) = 1.53$, $p = 0.25$, and the interaction were not significant, $F(3,14) = 1.60$, $p = 0.23$. The univariate tests suggest that the overall significant effect was due to a significant effect of time for the *math value for robotics* subscale, $F(1,16) = 14.78$, $p < 0.01$, $\eta^2 = 0.48$. The overall scale and the other two subscales were not significant. However, the only mean differences from pre to post that were significantly different from zero were a negative change within the *Calculational* condition for the *robotics interest* subscale ($M = -0.34$, 95% CI [-0.76, 0.07], $p = .10$) and a positive change also within the *Calculational* condition but for the *math value for robotics* subscale ($M = 0.66$, 95% CI [0.27, 1.04]). This indicates that there was not enough evidence to conclude that participation in the *Mechanistic* condition had an impact on students' disciplinary engagement, but participation in the *Calculational* condition led to changes in engagement as both a decrease in interest in robotics and an increase in a sense of the importance of math for robotics. This pattern of changes in attitudes observed in the *Calculational* condition was the same pattern observed for in the *Model Eliciting* condition from Study 4.

Table 14. *Calculational* and *Mechanistic* groups attitudes outcomes results

Measure	Calculational (n = 7)				Mechanistic (n = 9)			
	Pre M (SD)	Post M (SD)	r	d	Pre M (SD)	Post M (SD)	r	d
Overall	0.3 (0.9)	0.4 (1.0)	0.9	0.0	1.0 (0.5)	1.0 (0.5)	0.8	0.1
Robotics Interest[a]	0.4 (1.1)	0.1 (1.4)	0.8	−0.3	1.2 (0.4)	1.2 (0.5)	0.7	0.0
Math Interest	−0.1 (1.5)	−0.4 (1.5)	0.8	−0.2	0.7 (0.8)	0.6 (0.7)	0.6	−0.2
Math Value for Robotics	0.7 (0.8)	1.3 (0.5)	0.6	1.1	1.1 (0.7)	1.4 (0.6)	0.8	0.5

[a] No inferential statistics were conducted on the *robotics interest* data for the *Mechanistic* group due to the low level of internal consistency for that subscale in that sample.

[+] $p < .10$. * $p < .05$. ** $p < .01$. *** $p < .001$.

3.4.2.3 Student work

Analyses of the whole class discussion posters and talk suggest that math was central to the activity of both groups, but they connected math to the situation in substantively different ways. All posters from both groups included explicit numerical operations. No posters in either group used an entirely guess-and-test strategy (although some had guess-and-check components within their overall strategy). Taken together, each of these is an indication that overall both groups engaged directly with the connection between math in robots.
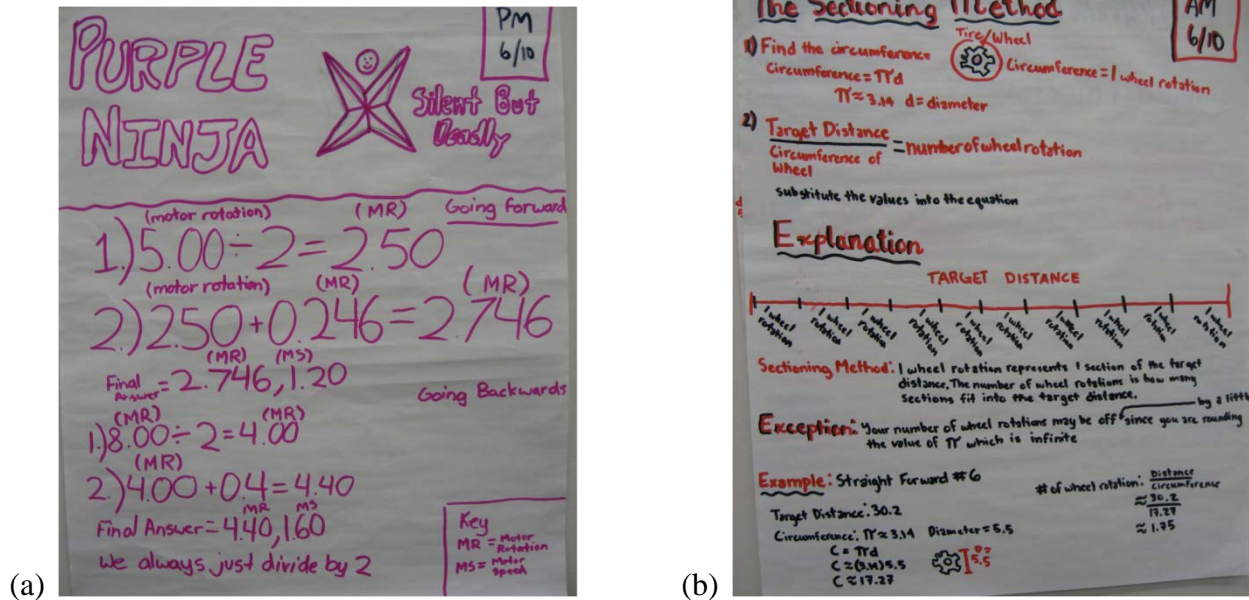
Figure 24. Typical (a) *Calculational* versus (b) *Mechanistic* posters

3.4.2.4 Posters

Examples of typical posters generated by the contrasting groups are provided in Figure 24. The *Calculational* poster (Figure 24a) is typical of that group in that it includes more numbers and less pictures suggesting an emphasis on mathematics of that form. The *Mechanistic* poster (Figure 24b) is typical of that group in that it includes more verbal explanations and pictures (e.g., the total distance partitioned into 1-wheel-rotation segments).

A coding scheme was developed to separately code the quality level of a poster from the level of incorporation of mechanistic thinking. The mechanistic code served as a manipulation check to determine whether the *Mechanistic* group was thinking about the situation differently than the *Calculational* group. It was proposed that this coding would be independent of quality so as to be a fair comparison across both conditions. There were 4 codes and each were coded as a 1 = present or 0 = absent:

(a) M1 – Physical Features

The solution uses a physical feature of the robot (wheel size, track width) to connect program parameters (motor rotations, motor speed) with robot movements (distance, angle turned, time), rather than an empirical finding or association based only on the direct connection between program parameters and robot movements.

(b) M2 – Labels Intermediate Values

Each of the intermediate values that are calculated in the strategy has referents in the situation that are explicitly defined in the poster either in words or in pictures.

(c) M3 – Includes Situation Pictures

Includes pictures that illustrate the entities and/or activities that are directly relevant to the strategy. The picture does not count if it illustrates a metaphor or superficial association with the situation.

(d) M4 – Includes an Explanation

Uses words that address why the strategy works using aspect of the situation or general knowledge beyond the effect itself (i.e., that it matches the data or does the right thing).

Table 15 displays the results from the mechanistic coding scheme. The *Mechanistic* condition was coded higher on all 4 dimensions indicating that indeed the students in the *Mechanistic* condition were thinking about the task more mechanistically than the students in the *Calculational* condition. The biggest differences between conditions were that the *Mechanistic* condition was more likely to include physical features of the robots in their strategies and to use situational pictures to illustrate their ideas.

Table 15. Mechanistic score of the group posters

Measure	*Calculational*	*Mechanistic*
Physical Features	0	6
Intermediate Values	8	12
Situation Pictures	1	7
Explanations	4	8

A separate coding scheme was developed to code the quality level of a poster. There were 4 codes and each were coded as a 1 = present or 0 = absent:

(a) Q1 – Valid

The strategy will get a reasonably accurate answer every time for the level of the situation that it is addressing. The answer does not have to be exact as long as it is within some reasonable range.

(b) Q2 – Steps Clear

The steps required to implement the strategy are well-defined such that another person could carry them out (even if some guessing would be involved).

(c) Q3 – Fully Specified

All of the used quantities can be calculated/measured; none of the used quantities require guessing or some sort of unspecified "adjustment". It is not required that they specify exactly how to calculate/measure the quantities as long as they recognize each as calculable/measurable.

(d) Q4 – Generalized

There is some explicit representation (words or pictures) of how the relevant quantities would be found using any robot (or pair of robots).

Table 16 displays the results from the mechanistic coding scheme. For this coding scheme both conditions developed similar quality posters in terms of using valid strategies and being clear about the steps in their strategies. This is evidence that *Calculational* condition did engage with the unit at a high level. Where the conditions differed, however, was on the fully-specified and generalized dimensions. This indicates that the *Calculational* condition was more likely to use adjustments, which is consistent with more of an answer-focus, or guessing, and with not being able to fully capture the data numerically. Similarly, the generalized dimension is harder to do when thinking about the situation numerically rather than mechanistically.

Table 16. Quality score of the group posters

Measure	*Calculational*	*Mechanistic*
Valid	13	13
Clear Steps	15	15
Fully Specified	6	15
Generalized	8	11

3.4.2.5   High level mathematics in the *Calculational* group

A possible explanation for the differences between groups may be that the *Calculational* group engaged in only low-level procedural work rather than engaging with the task at a deep level. On the contrary, qualitative observations of the *Calculational* group whole-class discussions did include high-level talk about mathematics that included: (1) connecting math ideas to the situation, and (2) building off each other's ideas to find more explicit, general solutions.

In this example, Seth, from one of the *Calculational* teams, is trying to justify to the rest of the group why his strategy of taking the ratio of the times is a more precise method for adjusting to synchronize the timing than just dividing by two:

262.	Scott:	Why do you do that again?
263.	Mr. E:	Well, first, do we understand what he is dividing? I agree that there is also a why question. But first do we understand, so Seth is suggesting, you take the correct time, which is the time that Beyonce does, that's what you're supposed to do for the dance routine, you divide that by the time that, um, Justin took
264.	Scott:	To get there
265.	Mr. E:	When he was going to fast.
266.	Scott:	Okay.
267.	Mr. E:	And now the question is why. Why would you do that? Frank, this is a question for you too. We are trying to figure out what he did, why it makes sense.
268.	Seth:	It's showing the, um, like how, sort of like how the Green team had, divided by two, but we wanted it more exact number, which is gives us the, the exact, um, which gives us the more exact number of how much the time, of how much the speed is. It's a bit less than half the time.

In line 268 Seth articulates both how his strategy connects with and extends the Green team's strategy of dividing by two, and he connects his operation to the quantities in the situation by suggesting how to interpret the ratio as representing "a bit less than half the time." Examples of both justifying and connecting can be found throughout the discussions. On the other hand, as predicted by the manipulation, the justifications used in the *Calculational* did not use physical mechanisms. Instead, they simply described number patterns and the inputs and outputs (in this case "time" is the output).

Another interesting aspect of the *Calculational* groups solutions that emerged in that group but not in the *Mechanistic* group was that they developed language for describing patterns that they couldn't explain. The numerical strategies in the *Calculational* group often became complex and hard to follow. As a result, they developed the term "too smart" to indicate when a team did a calculation that was too difficult to follow. I argue that these instances of not being able to explain their reasoning or to understand some one else's reasoning were not because the individuals in this group lacked the mathematical sophistication. On the contrary, they did capture and explain some very complex patterns. Instead, I argue that they were limited by framing the task entirely as a calculational one, which led to solution strategies that were unnecessarily complex for capturing the essential elements of the situation.

3.4.2.6 Post-interviews and the competition task

The post reflection interviews were conduced by a colleague of the author so as not bias students responses. This interview was conducted with each team at the end of the last day (see Appendix C.3 for the list of questions). One of the questions asked the teams whether they used the strategies for robot movements that they had invented during the week on the competition task. If the team responded affirmatively, then that would be an indication of their recognition that both the robot dancing and competition tasks are structurally similar and both would benefit from a math-based strategy.

All four *Mechanistic* groups but only one *Calculational* group reported using the invented RSD strategies in the design competition task. For example, here is a response from a student in one of the *Calculational* groups when asked whether they used any of the strategies they developed during the week in the RSD activities on the competition task:

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| 269. | S: | Not really. No. Cause there isn't any, like, it isn't like we are comparing two different robots to do the same thing. All robots are the same in this. We're not using two different robots to do the same thing. So there really is no need for any strategies like that. |

<div align="right">*(Calculational Red Team)*</div>

A striking aspect to this student's response is that he sees the RSD situation and the competition task situation as being different in ways that make the methods unusable in one versus the other. A similar distinction between the tasks, but for a different reason, was made by another *Calculational* group:

270.	S1:	Cause it's a different robot. It has bigger wheels.
271.	S2:	Well, we don't know like, I don't really know why we didn't use one of our strategies. We just decided to use one and didn't really think about the others.
272.	S1:	We're still in the lead.
273.	I:	So it's working for you?
274.	S1, S2:	Yeah

<div align="right">*(Calculational Purple Team)*</div>

In contrast, all of the *Mechanistic* groups responded that they did use strategies they developed during the week in the RSD tasks when developing their solution for the competition task. For example:

275.	S1:	We sort of first wanted to find the distance of where it really had to go.
276.	S2:	So we measured them and divided it and we got the distance.
277.	S1:	The wheel rotations.
278.	S2:	Yeah, the wheel rotations.
279.	S1:	Of how far it was supposed to go.

280.   …

281.   S3:   Oh when we got the circumference of the wheel, I thought it would be easier just to measure the thing in centimeters. But everything else was in inches. So I just got the centimeters and divided by 2.5 cause there's 2.5 centimeters in an inch. And that's how we got 10.4 as a circumference.

(Mechanistic Red Team)

Not only does this team see the connection and apply the same strategies, but also they recognize that that the same fundamental ideas are relevant and useful even though some aspects may need to be adapted (i.e., the unit of measure for length). Here is a second example from a *Mechanistic* group:

282.   S1:   We used the, the strategies that we learned all throughout the week. Um, we, like, for the straights, we, um, used the circumference of the wheel as the rotations and measured it, measured the area.

283.   I: What do you mean by measured the area?

284.   S2:   Like how far it was from here to here. And then we like said, I think the wheel was 26 cm, so we said one rotation would be 26 cm, two would be whatever that is times two.

(Mechanistic Purple Team)

As illustrated in this response, the students in the *Mechanistic* group still do have ideas that are not correct (i.e., measuring "area" rather than "distance" or "length"), but they nevertheless are able to focus on the key aspects that are structurally similar and to apply them appropriately in this competition task.

3.4.3 Discussion

Overall only the *Mechanistic* group significantly improved their problem solving from pre to post, whereas the *Calculational* group did not. In both conditions, students used and reasoned about mathematics in sensible ways that were well connected to the robot situation. As a result, both invented valid strategies that were useful in solving the RSD task.

As was expected given that the RSD unit was used in the *Calculational* condition of this study in the same form that it was used in Study 4, both the *Model Eliciting* condition from Study 4 and the *Calculational* condition from this study exhibited similar engagement outcomes. Both groups had decreased interest in robotics as a result of participating in the RSD unit but more positive views about the value of math for robotics. The decreased interest in robotics is consistent with the explanation that engaging in rigorous mathematics within a robot activity—as exemplified by the RSD activity—is hard work and so may negatively affect students' interest if they originally thought of robotics as a purely fun and informal activity. The findings that both the *Calculational* condition from Study 6 and the *Model Eliciting* condition from Study 4 exhibited similar engagement outcomes along with the finding that so few teams within both of those conditions exhibited evidence of mechanistic reasoning in their posters suggests that the calculational orientation may be the default, dominant, or most common approach for connecting math with robotics in this situation. Students seem to need further, more explicit supports—as provided in the *Mechanistic* condition from this study—to take on the mechanistic approach. That the calculational orientation is the dominant orientation is consistent with Thompson et al. (1994) who note that moving out of the calculational orientation is very difficult to do, and even if one does manage to move out of that orientation for a little while, it is very easy to fall back

into it. The calculational orientation may have advantages in situations of increasing complexity (Koedinger et al., 2008), and so students' reliance on that strategy may be sensible and adaptive. Nevertheless, in this context a mechanistic approach seemed to be more productive.

Despite the difficulty in moving out of a calculational orientation, the students who participated in the *Mechanistic* group did to some extent take on a mechanistic orientation in the RSD unit as evidenced by their higher mechanistic scores on their posters. They used more explanations and situational pictures on their posters, used more intermediate values with clear references in the situation, and used more physical features of the robot as the basis for their strategies. All of these aspects suggest that it is possible to get students to take on this orientation, even if it was only for a short time in this setting. In addition, however, the *Mechanistic* group also was more likely to extend their experiences from the RSD unit to the competition task. All four *Mechanistic* teams did this even though the task was setup to make the correspondence between the RSD tasks and the competition seem distant. The *Calculational* group teams did not extend their experiences in the same way.

In addition to providing instructional opportunities for students to connect math to their robot activity, alternative ways to setup the activities influence how students approach the task and can be consequential for learning. As other researchers have argued (Greeno, 2009) and investigated empirically (Elby, 2001; Hutchison & Hammer, 2009; May & Etkina, 2002; Redish & Hammer, 2009), epistemological framing is an important factor. The studies reported in this second part add to that research base and provide evidence that even when students are able to make connections between mathematics and robotics, the particular math-to-robots approach that students take may lead to different impacts on learning both quantitatively and qualitatively.

4.0   GENERAL DISCUSSION AND CONCLUSIONS

4.1   SUMMARY OF RESULTS

I am now in a position to update the figure from the end of Part 1 (Figure 19) by adding in the *Calculational* and *Mechanistic* groups from the model-eliciting environment in Study 6 (Figure 25). With this figure as a guide, it is now possible to consider the findings of this dissertation project as a whole. Although in many cases the low number of students within each learning environment made it difficult to reliably assess the impact of participation, when the quantitative results are combined with qualitative results of students' work and their reflections, then it is possible to draw conclusions. One notable result is that there is a wide range of possibilities for approaching introductory robotics, from the formal classroom unit (Study 1) to the informal competition settings (Study 3). The versions of the RSD unit (Study 2, Study 4, and Study 6) attempted to bridge between these extremes. Further, within each of these possibilities for instructional design of the learning environment, there were further possibilities for different approaches by the students within them in how they used math as a tool for problem solving.
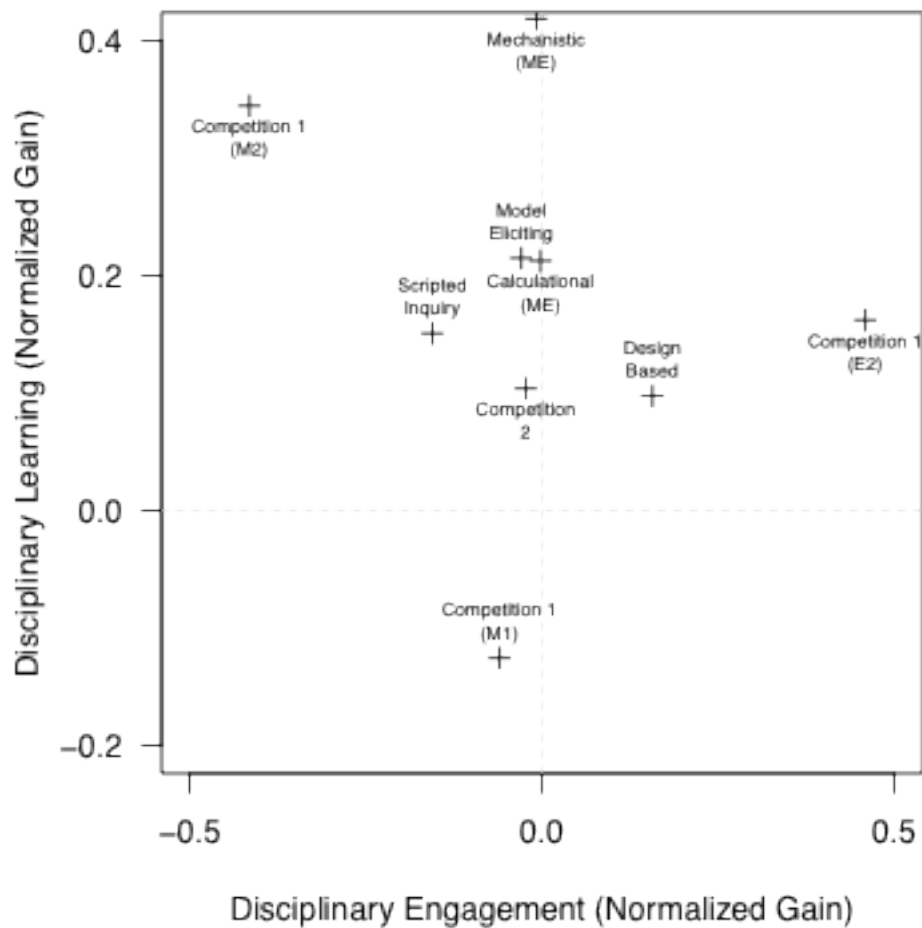
Figure 25. Effects of the environments and framings on learning and engagement

In the *Scripted Inquiry* environment (Study 1) the students did mathematics in very explicit ways, but the connections to using the math to understand and program robot movements were mostly implicit. As a result, the improvement of students in problem solving was mostly limited to math outside of the robot context, their interest in math decreased, and their sense of what math is useful for was narrow and limited to its use as a calculational tool. In the *Competition* environment (Study 3), each team approached solving the challenge in very different ways, with only a quarter of the teams choosing to use math in their solutions.

Interestingly, the teams that used math either did so extremely effectively in the competition or very poorly, suggesting that using math is a risky strategy, but can have real benefits if done well. It was also the case that both *Focus Teams* that chose to use math (Team E2 and Team M2) were on the positive side of the disciplinary learning scale in Figure 25, but the *Focus Team* that didn't use math (Team M1) was on the negative side. Another notable finding is that even though the *Competition* environment is generally considered a fun activity, none of the *Focus Teams*, with the possible exception of the elementary school team (Team E2), had increases in engagement. This surprising finding suggests that the relationship between the nature of the robot learning environment and its effect on interest in robots, on interest in math, and on views about their connections is not straightforward.

Turning to the designed learning environments, it proved to be a challenge to design a structured set of activities that focused students' attention effectively on the connections between math and robots in an engaging way. Although the *Design Based* environment (Study 2) demonstrated a promising approach for getting students to begin to make those connections to math in ways that were useful within the robot context, further support and focus was required to help students make more substantive progress. The revision of the *Design Based* environment as a *Model Eliciting* environment (Study 4) served to help students more immediately focus on connecting to math within the robot dancing activity from the beginning of the unit activities. Students were engaged by the challenge of the synchronization problem itself, not just programming robots to dance, and developed sophisticated math ideas that took advantage of proportional reasoning as a tool for solving that synchronization problem. Although this did lead to improvements in problem solving and in ratings about the value of math in robotics, it also had the less desirable effect of decreasing students' interest in robotics. As expected given that

they were the same unit, in Figure 25 the *Calculational* group from Study 6 is clustered right next to the *Model Eliciting* group from Study 4, providing further support for the claim that the model-eliciting environment improves students ability to connect math in robotics while maintaining overall levels of disciplinary engagement. Again, however, the pattern in both instances within the disciplinary engagement subscales is that students increase in their sense of the value of math for doing robotics, suggesting that they have come to appreciate the math as a tool in this situation. But the pattern is also that that desirable positive effect on valuing math in robotics occurs concurrently with a less desirable decrease in interest in robotics. Although this decrease in robotics interest is concerning, it is worth noting that in both cases students overall robotics interest was still positive even at post. I will consider possible explanations for this decrease in robotics interest further in the following section.

Continuing to build on the learning environment design, the *Mechanistic* group from Study 6 represents another revision of the RSD unit that seemed to help students make even deeper connections to math in their robot activities. By focusing on using math not only as a calculational tool for manipulating numbers and finding answers, but also as a representational tool for connecting to intuitions about how the structures of the robot function and behave in the situation, students were able to improve even more on their problem solving. In addition to their significant gains in learning on the pre-post problem solving assessment, this improvement in problem solving was also evident in how the nature of their solutions to the RSD problem relied on more physical mechanisms and images of those mechanisms, in how the substance of their RSD solutions didn't use guessing and were often generalized to other robots, and in their ability to connect their RSD solutions to a competition task. Also notable in the *Mechanistic* group was that there was not the same decrease in robotics interest that was observed in the other *Model*

Eliciting groups (the *Model Eliciting* group from Study 4 and the *Calculational* group from Study 6). That the *Mechanistic* group maintained their interest in robotics may mean that approaching the rigorous task of connecting math to robotics mechanistically may not only be better for learning, but also for engagement as well.

4.2   MODELING AND MECHANISTIC THINKING

4.2.1   Empowering beginning roboticists as systematic designers

The research presented in this dissertation project provides support to the idea that math can provide an organization to the complexity of situations, which may in turn lead to better understanding and problem solving. Utilizing quantitative representations of qualitative features of situations (Thompson, 1994) enables more precise observations and systematic manipulation of those features. In the case of robots, the motor rotations used to program the robot to move are already in a quantitative form in the programming environment, which means that students may already be encouraged by the situation to use math. And yet, in the *Competition* environment from Study 3, most teams don't use math. Only students that take the additional step of quantifying the distance the robot moves are able to more systematically approach that problem and develop solutions that work efficiently and accurately across different moves. Similarly, the students in the *Scripted Inquiry* environment from Study 1 are very attuned to the use of math as an efficient way to get the answers when working with the robots and as an alternative to guessing.

In addition to efficiency, however, math has the additional advantage of providing an organization for students to more effectively incorporate more than one feature of the system into their thinking and designing at the same time. Schwartz et al. (2005) explain that this advantage of using math may be the result of converting different perceptual qualities of the system into the same ontology—number—such that now those features can be compared and related. The students in the *Model Eliciting* environments were able to use math in this way to invent solutions that related multiple aspects of the system in predicting and controlling the robots' movements. Integrating multiple aspects of the system together such that they are considered simultaneously is considered to be very sophisticated understanding in the balance scale tasks as used in Schwartz et al. (2005), but the integration of multiple features is also key in other tasks and contexts (Siegler & Chen, 2008). Further, understanding the causal effect of individual variables may be necessary but not sufficient for performing well at multivariable prediction tasks that require coordinating between the variables (Kuhn, 2007). Although providing practice with additive and interactive effects of variables in a system along with explicit instruction about the logic of models for analyzing multiple variable systems may be beneficial (Kuhn, Pease, & Wirkala, 2009), it may be that students' difficulties are also the result of not having effective tools for managing the cognitive load required to think about those variables simultaneously. Mathematics may provide one option for making such complex thinking more tractable as abstract representations do provide advantages over concrete representations when the complexity of the problem situation increases (Koedinger et al., 2008). Although the students in the studies reported here may not have reached the level of math use as a tool for thinking about inaccessible phenomena as do professional engineers (Gainsburg, 2006), their use of math did go beyond a mere computational tool for making the solution of simple, straightforward problems

more efficient. The students in the *Model Eliciting* groups also utilized math as a thinking tool for helping them to understand and model the complex relations and behaviors within the situation. This use of math helped them to invent sophisticated strategies for controlling the movements of their robots that were useful and efficient for coordinating multiple aspects of the situation in solving their immediate problems, but their solutions were also general and adaptable to different movements and robots.

4.2.2   Concreteness versus abstractness in math-to-robot connections

Given that the students in the *Model Eliciting* groups did reach a more sophisticated understanding of the connections between math and robots in this context, there are questions about the extent to which their understanding was dependent upon knowledge of robots, of wheels and rotations, and of the particular movement mechanisms involved in this situation. That the students in the *Mechanistic* group from Study 6 were more likely to transfer their strategies to the competition task than the students in the *Calculational* group suggests that the students in the *Mechanistic* group developed strategies with a broader set of conditions of applicability. But there is not evidence to suggest the students in the *Mechanistic* group developed strategies or understandings that would generalize to even broader contexts involving other non-robot proportional situations.

The local conceptual development (Lesh & Harel, 2003) in which the *Model Eliciting* students engaged—wherein they essentially invented their own contextually-embedded versions of the generally applicable math concept of proportions—would very possibly not transfer spontaneously to non-robot contexts. This seems reasonable given that there was little explicit

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instruction or opportunity to develop strategies and make connections outside of the robot context (Norton, 2006). However, one possible result was that the *Mechanistic* group developed understandings that would have greater potential to transfer outside of the robot context as a result of subsequent learning experiences than would students in the other learning environments. In other words, it may be that the *Mechanistic* groups were better prepared for future learning (Bransford & Schwartz, 1999) as a result of developing deeper understandings within this initial robot movement context. Research on concreteness versus abstractness suggests that progressive idealization or "concreteness fading" has the greatest transferability, compared to abstract to concrete sequences and abstract or concrete experiences alone (Goldstone & Son, 2005). The originally grounded and well-understood ideas can over time become less tied to their specific contexts, but still serve as a rich source of conceptual and strategic knowledge from which to build more abstract and generalizable knowledge. Certainly there is value in optimizing the initial contextualized learning experience—such that the concrete ideas that students learn in that situation are indeed well understood and locally adaptable—prior to or in conjunction with optimizing the features of the learning environment that are intended to promote transfer outside of that context, even though both the initial context and the transfer context may be ultimately important.

4.2.3   Epistemological frames for connecting math to robots

In addition to transfer of the particular concepts and strategies used in the robot context to other robot and non-robot contexts, another form of transfer may be possible. Students in the *Model Eliciting* units may have learned epistemological notions about what sorts of resources are best

drawn on in higher quality technological problem solving. The most relevant result supporting the claim that students' epistemological framing of robot tasks did change was the consistent increases in students' sense of the value of math for robotics from pre to post across the RSD implementations. The one exception to that was in the *Mechanistic* group in Study 6, wherein a significant change in the value of math for robotics was not detected. However, even in this group there was a positive trend from pre to post, their post ratings were still very positive in an absolute sense, and they all transferred the math strategies they created in the RSD unit to the final competition task. Taken together, this evidence suggest that the *Mechanistic* group—in addition to the other RSD groups—saw the math as a useful resource for solving robot problems. However, that evidence is not directly relevant to the question of whether these students would be likely to transfer their revised math-to-robot epistemological frames to non-robot contexts.

Similar to proportionality as a general mathematical principle, a likely scenario is that the students wouldn't necessarily transfer a mechanistic epistemological frame spontaneously, but would be better prepared to do so if provided with more explicit support in a new context. More specifically, providing students with explicit support for reflection on how it was useful to connect their intuitions about mechanisms with mathematical representations might have made it possible that students would approach a new problem in a similar manner. One approach to facilitating epistemological development is to have students engage in introductory activities in content-light contexts that make the "rules" and criteria of scientific epistemologies salient prior to engaging in more content-rich inquiries (Cartier, Passmore, Stewart, & Willauer, 2005). Prior to doing a unit about modeling celestial motion, to make explicit what is meant by scientific explanations, Cartier et al. (2005) had students participate in a black box activity in which students poured water in the top of a black box, observed patterns of water that came out the

bottom, and created and justified models about mechanisms within the box that might account for the pattern. Although successful in that case, it may be that it is neither necessary nor optimal to have students participate in extra activities to setup the target activities if the epistemological supports can be built directly into the target activities themselves.

An alternative approach might be to modify the tasks within the core context such that the epistemological aspects are not simply a prescribed way of doing things, but rather prove to be a more useful approach within the context that students and the class value directly. For example, in a semester-long undergraduate introductory physics course for biology majors, Redish and Hammer (2009) designed the course to explicitly encourage students to develop productive epistemologies about physics as a refinement of everyday thinking. They used vocabulary that made different epistemological frames for their class discussions explicit, such as "shopping for ideas" and "playing the implications game." Redish and Hammer also modified both peer instruction and interactive lecture demonstrations with the goal that students would not just focus on the answer and set aside their intuitions, but would instead recognize the productive aspects of their intuitions and how those intuitions relate to the canonical answers. In this example, the epistemological frames were developed over the course of an entire semester, and so it may be that meaningful, transferable changes in the epistemologies that students choose to draw from can only be achieved over time and after explicit support for using those alternative, more productive epistemologies in multiple instances. However, adding explicit vocabulary and revising the RSD activities to directly facilitate epistemological change with a focus on mechanistic approaches would likely increase the chance that the students would transfer their approach to non-robot contexts outside of the RSD unit.

4.3   DESIGNING LEARNING ENVIRONMENTS

4.3.1   Targeting the connections between math and robots

A major issue analyzed for each of the learning environments studied in this dissertation was the extent to which the environment facilitated students in using math as a tool for understanding and problem solving with the robots. What was clear from the analyses was that there were many different ways to introduce students to robotics, ranging from the highly structured *Scripted Inquiry* unit to the open-ended *Competition*. Each of these learning environments makes the connections between robots and math more or less salient in the activity that students do and in the work that they produce. Although perhaps not surprising, one result is that when left mostly unstructured, students are likely to invent problem solutions that don't include math, as is the case for most teams observed in the *Competition*. Thus, even if there is the potential to mathematize a situation, students may not see the value in doing so and may find creative ways to approach the problem without math. Conversely, the observations of the *Scripted Inquiry* environment suggested that overly structuring the activity such that the math is essentially all that students do even though their activity takes place in the context of robots leads to learning mathematics disconnected from actual problem solving with robots. As a result, aligning mathematics and robotics in a learning environment is not simply about putting students in a situation where the math is remotely possible or strictly necessary. Rather, aligning math in robot activities is complex. It requires subtle shifts in how problems are introduced and supported such that the math is motivated as a tool for understanding and problem solving and students attend to the connections between robots and math rather than either alone.

In their learning-goals-driven design model for developing project-based curricula, Krajcik, McNeill, and Reiser (2008) articulate how the process of aligning instructional tasks to learning goals is necessarily iterative and must take place at a fine-grained level in order to move beyond superficial, topic-level connections. Krajcik et al. describe how their instructional designs are revised by taking into account feedback from external reviewers and observations of their tasks enacted in classrooms, since theoretical designs are often enacted in ways unintended or unanticipated by the instructional designers. Furthermore, Krajcik et al. view alignment not as a binary criterion, but instead recognize degrees of alignment and work toward obtaining the "sufficient alignment" needed such that the knowledge they are targeting is both necessary and sufficient for their desired student learning performances. The idea of alignment helps focus the data collected and analysis undertaken so that the revisions to the instructional designs are targeted and more likely to ultimately be effective. In the case of the RSD unit, the revisions of the unit activities over the course of the studies helped the unit as a whole to more directly target the connections between math and robots with the mathematics serving as a tool for understanding and problem solving in the robot context.

In elaborating the principles for the design of model eliciting activities, Lesh et al. (Lesh et al., 2000) have also recognized the complexity and subtlety of setting up situations such that the mathematics is strongly motivated as a useful tool. In their case, they document how teachers modified the setup of a model eliciting activity in small but significant ways over three iterations so that students increasingly thought that the problem to be solved was less about making a one-time solution and more about inventing a general tool for thinking about the situation. In the RSD unit a similar set of revisions were made so that almost immediately after being introduced to the problem students attended not just to making robots dance or to making a single

synchronized dance, but rather to inventing a tool for thinking about synchronizing the movements of different robots. A useful design criterion that emerged through reflections on the RSD unit revisions was that the more immediately students encoded the problem as being about math-to-robot connections the better. The time students spent working on designing their own dance routines in the *Design Based* version of the RSD unit didn't align strongly with making math-to-robot connections. In addition, that part of the unit likely influenced students' framing of the second part on synchronization, even though the second part was better aligned with math-to-robot connections. The initial activities in the first part encouraged students to think of the overall goal of the unit as being to design a single, working synchronized dance rather than to design a general and adaptable toolkit for making synchronized movements. Overall, the analysis of the design of the RSD unit provides further support to the usefulness of alignment as a concept to help focus design iterations and to the importance of analyzing enactments of the design. The goal of the revisions is to ensure that what students attend to in the unit aligns strongly and immediately with the target ideas at a fine-grained level.

4.3.2 Targeting engagement and learning

A second major issue analyzed for each of the learning environments studied in this dissertation was the extent to which the outcomes observed with students consisted of positive changes on measures both of learning and of engagement. In some cases, as in the *Scripted Inquiry* environment in Study 1, some learning did occur, but at the expense of students' interest in mathematics and overall engagement. Such negative experiences are likely to discourage students to pursue further study in robotics or other STEM-related fields. In the *Competition*

cases, few changes in engagement were observed, which may be the result of those select participants already having high levels of engagement that are for most part maintained or of the low levels of participating students in each case, which made a change difficult to reliably detect. A more puzzling finding was the *Model Eliciting* case, in which students' did improve in their problem solving and increased their sense of the value of math for robotics, but decreased their interest in robotics (even though their absolute levels were still positive). Although it is not clear whether outcomes like this would in fact discourage students to pursue further study in robotics or other STEM-related fields, it is worth trying to better understand what about the design of the *Model Eliciting* environments may have contributed to this result.

One possibility was the extent to which the learning environment design provided a personally interesting context for the robot problems. Observations within the *Design Based* environments suggested that indeed having students create their own dance routines was engaging for students. However, the issue with that design approach arose when the students transitioned to synchronizing their routines across different robots. The personalization aspect was no longer central and so relative to the first part of the unit the tasks became less engaging. In addition, because each team had created such different dance routines it was difficult to compare between them when they presented their synchronization ideas to the rest of the teams using data from their individual routines. Thus, the personalization may have had some initial engagement benefits, but on balance may have limited learning and engagement in the synchronization activities. Son and Goldstone (2009) have demonstrated how personally-relevant framings of tasks can encourage students to take on goals that are specific to an individual's point of view and so may limit students' attention to the underlying general structure common across instances and contexts. In that sense, there may be negative consequences to learning that

result from personalizing the context. The *Model Eliciting* versions of the RSD unit in both Study 4 and Study 6 did not have the students make their own dance routines and obtained better learning results as a consequence. And yet, personalization may have some benefits if that initial engagement were somehow better focused on the problem that aligned with the learning goals.

Further exploration of the influence of personalization suggests that from a cognitive perspective personalization may not be beneficial overall, but may have certain limited benefits that are worth teasing apart. Walkington and Maull (2011) provide data that suggest that personalization can assist students in problem solving by making them more efficient at easier parts of the problem such as reading and understanding the problem and identifying the givens and unknowns. This allows students to focus more of their cognitive resources on the difficult parts of the problem, and in doing so they are likely to be more successful overall. The RSD unit attempted to include personalization in the learning environment design more in this way, by setting up the problem in a way that students would have an implicit grasp of the issues almost immediately. By framing the problem about dancing out of sync with perceptually salient differences between the robots' movements, students were able to attend more specifically to the synchronization problem while also activating their own cognitive resources of mechanisms and of mathematics to solve the problem. Observations of the students being introduced to the *Model Eliciting* version of the problem when first seeing the robots dancing out of sync with each other suggested that that problem itself was engaging to students and they felt motivated to solve it. The observed engagement in the revised RSD task suggests that personalization may have an important role in the design of learning environments, but that instructional designers should be aware of how that personalization assists rather than hinders students in attending to the underlying general structure of the problem.

4.4   CONCLUSION

Although each revision of the RSD unit has come closer, the ideal of identifying the features of learning environments that reach the level of "hard fun" has not yet been realized as a result of this dissertation project. Learning environment designs that align immediately and strongly to the connections between robots and math and that use personalization to assist students in attending to the cognitively challenging aspects of the problem are useful starting points. But more research is needed to optimize the learning environment designs such that students' productively explore the ideas at deeper levels, are increasingly motivated and interested in doing so, and sustain that productive disciplinary engagement over longer periods of time.

ATTITUDES SURVEY ITEMS

Item	Text
Robotics Interest	
RI2	I enjoy working on robotics problems
RI2	I would even give up some of my spare time to learn new topics in robotics
RI3	While working on a robotics problem, it sometimes happens that I don't notice time passing
RI4	Robotics is dull and boring *(reverse coded)*
Math Interest	
MI1	I enjoy working on mathematical problems
MI2	I would even give up some of my spare time to learn new topics in mathematics
MI3	While working on a mathematical problem, it sometimes happens that I don't notice time passing
MI4	Mathematics is dull and boring *(reverse coded)*

Item	Text
Math Value for Robotics	
MVR1	I can think of many ways to use math in robotics
MVR2	Mathematics helps teach a person to think about robotics
MVR3	I believe studying math helps me with problem solving in robotics
MVR4	Mathematics is sometimes useful but not that important in robotics *(reverse coded)*

APPENDIX B

PROBLEM SOLVING ASSESSMENT ITEMS

B.1   STUDY 1 – SCRIPTED INQUIRY ASSESSMENT ITEMS

B.1.1   Form A

1A-1.   How many miles will she run?

Debbie runs 0.6 of a mile every day. How many miles will Debbie run in 45 days?

1A-2.   What is the radius of the wheel?

If the diameter of a robot's wheel is 30 centimeters, what is the radius of the wheel?

    a.   180 cm
    b.   90 cm
    c.   60 cm
    d.   15 cm
    e.   10 cm

1A-3. How far forward did the robot travel?

Start   **End**

0   10   20   30   40
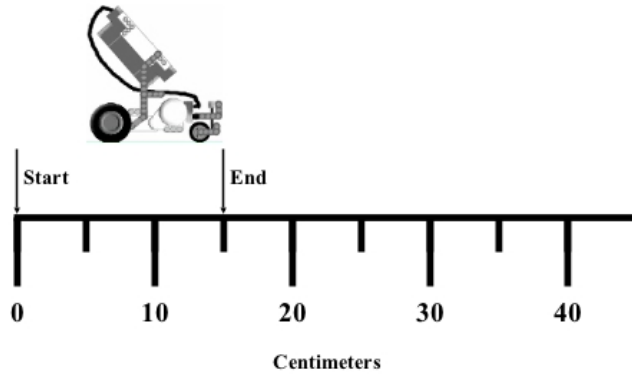
Centimeters

In the figure above, the robot started with its front at the 0 mark. How far forward has the robot traveled?

  a. 50 centimeters
  b. 40 centimeters
  c. 25 centimeters
  d. 15 centimeters
  e. 10 centimeters

1A-4. What is the estimated distance along her path?
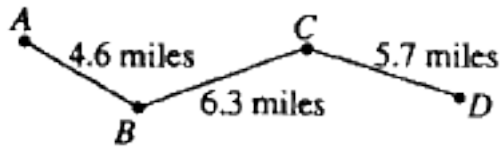
Carol wanted to estimate the distance from A to D along the path shown on the map above. She correctly rounded each of the given distances to the nearest mile and then added them. Which of the following sums could be hers?

  a. $4 + 6 + 5 = 15$
  b. $5 + 6 + 5 = 16$
  c. $5 + 6 + 6 = 17$
  d. $5 + 7 + 6 = 18$

1A-5. How many degrees did the robot turn?
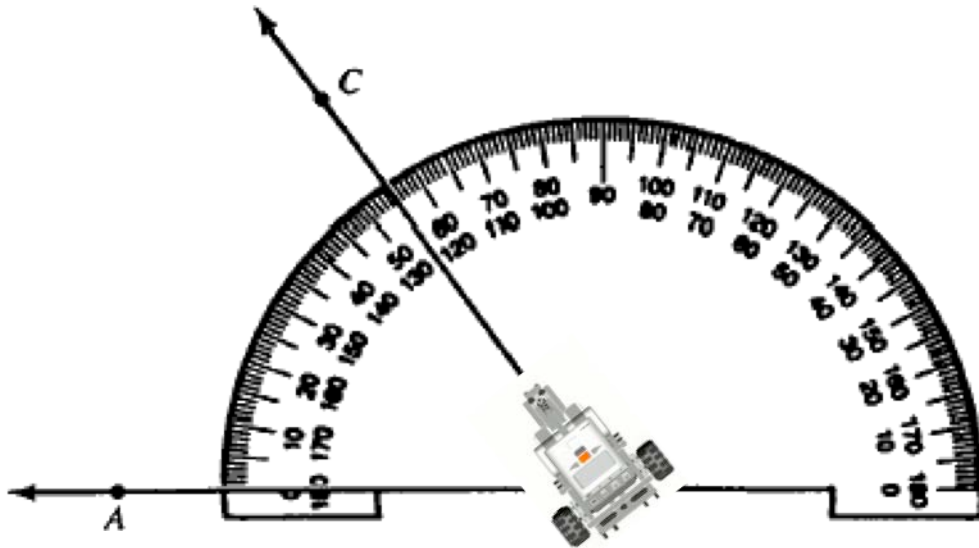
In the figure above, a robot started out facing point A and then made a point turn so that it now faces point C (as shown). How many degrees did the robot turn?

    a. 135°
    b. 125°
    c. 65°
    d. 60°
    e. 55°

1A-6. What is n?

If $\dfrac{2}{25} = \dfrac{n}{500}$, then $n =$

    a. 10
    b. 20
    c. 30
    d. 40
    e. 50

1A-7. Where is the distance on the line?

Kayla marked out the distances on a table to see how far her robot would travel forward when it started at 0. Her marks looked like the figure below. On the figure below, place a dot at the point that could represent a distance traveled of 1.75.
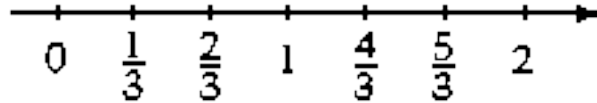
1A-8. What is the result when you divide?

Divide:

$$15\overline{)30.45}$$

1A-9. How many broken robots expected?

From a collection of 500 robots, a sample of 25 was selected at random and tested. If 2 robots in the sample were found to have broken sensors, how many robots with broken sensors would be expected in the entire collection?

a. 50
b. 40
c. 30
d. 20
e. 10

1A-10. How long will it take the machine?

A certain machine produces 300 nails per minute. At this rate, how long will it take the machine to produce enough nails to fill 5 boxes of nails if each box will contain 250 nails?

    a. 4 min
    b. 4 min 6 sec
    c. 4 min 10 sec
    d. 4 min 50 sec
    e. 5 min

Questions 11-13 refer to the following diagram. The diagram is part of a scale drawing of a robot maze.

1A-11. What is the length of the side of the maze?

What is the length, in feet, of the segment in the maze whose dimension is not given in the diagram?

    a. 40
    b. 36
    c. 30
    d. 24
    e. 12

1A-12. What is the scale of the diagram of the maze?

Use the ruler provided to find, in terms of inches and feet, what scale has been used to construct the diagram.

1A-13. What would be the new length of the side of the maze?

If you were to redraw the diagram using a scale of 3/4 inch = 10 feet, what would be the length of the side that is 48 feet?

    a. 12.0 in
    b. 7.5 in
    c. 5.6 in
    d. 3.6 in
    e. 3.0 in

1A-14. What is the ratio of eaters?

In a group of 1,200 adults, there are 300 vegetarians. What is the ratio of nonvegetarians to vegetarians in the group?

    a. 1 to 3
    b. 1 to 4
    c. 3 to 1
    d. 4 to 1
    e. 4 to 3

1A-15. What's the circumference of the wheel?

Using the centimeter ruler provided, find the circumference of the robot wheel above. (Use π = 3.14.)

_____ centimeters

1A-16. Where is the arc?

On the circle with center C shown below, use the protractor to locate and label a point B that creates an arc AB with measure 235°. Darken this arc.

180
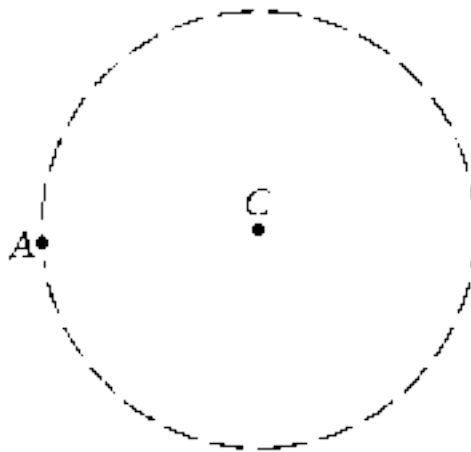
B.1.2 Form B

1B-1. How many centimeters did her robot move?

Jennifer's robot moves forward 0.6 centimeters every complete rotation of its wheels. How many centimeters will Jennifer's robot move forward in 45 complete wheel rotations?

1B-2. What is the radius of the circle?

If the diameter of a circle is 30 centimeters, what is the radius of the circle?

    a. 10 cm
    b. 15 cm
    c. 60 cm
    d. 90 cm
    e. 180 cm

1B-3.   How much liquid has been let out?
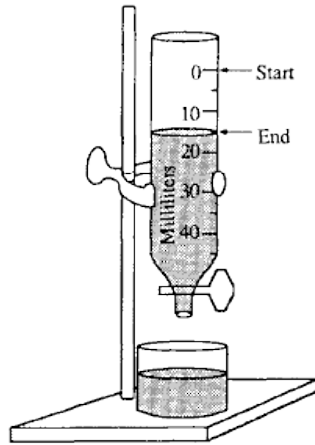
In the figure above, the tube was filled to the 0 mark at the start. How much liquid has been let out?

  a.   10 milliliters
  b.   15 milliliters
  c.   25 milliliters
  d.   40 milliliters
  e.   50 milliliters

1B-4.   What is the estimated distance the robot would travel on the path?
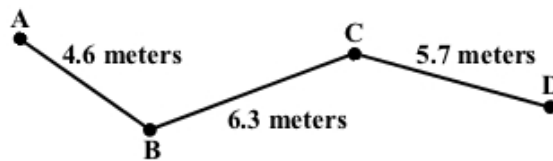
Marcus built a robot to do an obstacle course in his classroom. His robot is going to travel from A to D along the path shown on the map above. He wanted to estimate the distance that his robot would travel. Marcus correctly rounded each of the given distances to the nearest meter and then added them. Which of the following sums could be his?

  a. $5 + 7 + 6 = 18$
  b. $5 + 6 + 6 = 17$
  c. $5 + 6 + 5 = 16$
  d. $4 + 6 + 5 = 15$

1B-5. How many degrees is the angle?
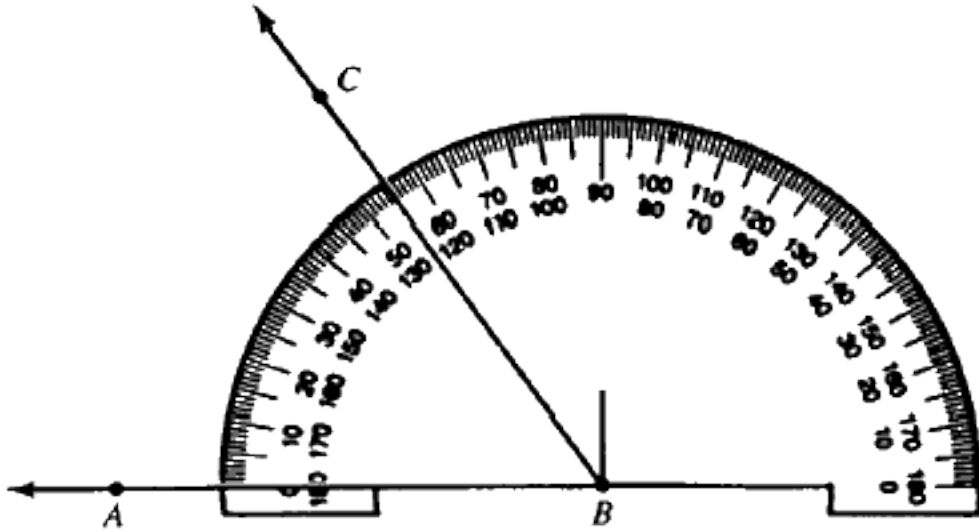
In the figure above, the measure of angle ABC is

    a. 55°
    b. 60°
    c. 65°
    d. 125°
    e. 135°

1B-6. What is the number of rotations?

If for every 2 complete rotations of the robot's wheels the robot moves forward 25 centimeters, how many rotations are needed to get the robot to move forward 500 centimeters?

    a. 50
    b. 40
    c. 30
    d. 20
    e. 10

1B-7. Where is the point on the line?

On the number line below, place a dot at the point that could represent 1.75.
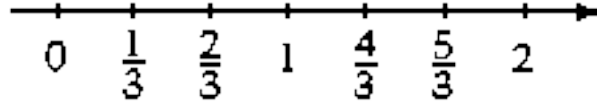
1B-8. How many meters for each rotation?

Greg programmed his robot to move forward 15 complete wheel rotations. His robot traveled forward a total of 30.45 meters. How many meters did Greg's robot move forward for each complete wheel rotation?

1B-9. How many dead batteries expected?

From a shipment of 500 batteries, a sample of 25 was selected at random and tested. If 2 batteries in the sample were found to be dead, how many dead batteries would be expected in the entire shipment?

    a. 10
    b. 20
    c. 30
    d. 40
    e. 50

1B-10. How long will it take the robot?

A vacuum-cleaning robot is cleaning the wood floor of a person's home at 300 square centimeters per minute. At this rate, how long will it take the vacuum-cleaning robot to clean enough of the floor to complete 5 rooms if each room has 250 square centimeters of floor?

    a. 5 min
    b. 4 min 50 sec
    c. 4 min 10 sec
    d. 4 min 6 sec
    e. 4 min

Questions 11-13 refer to the following diagram. The diagram is part of a scale drawing of a house.
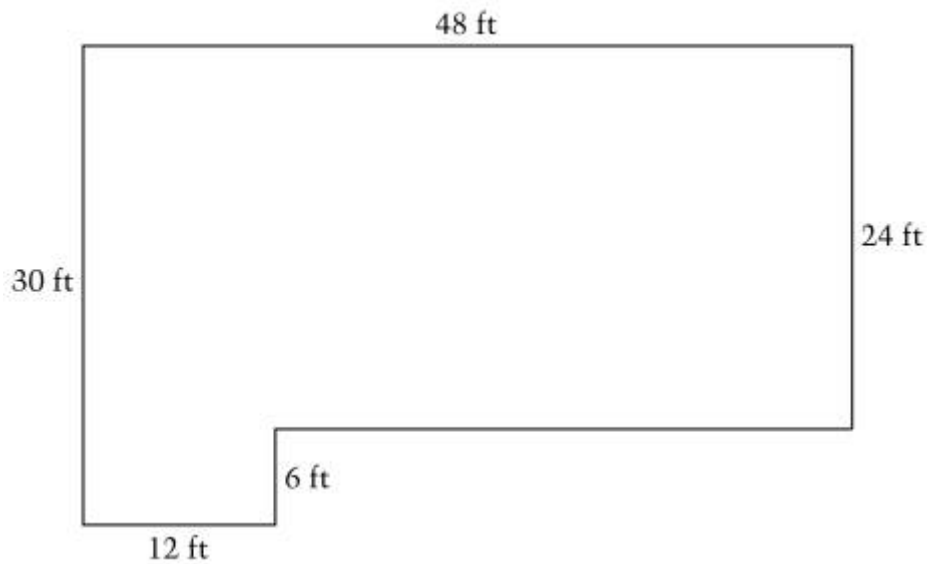
1B-11. What is the length of the side of the house?

What is the length, in feet, of the side whose dimension is not given in the diagram?

    a. 12
    b. 24
    c. 30
    d. 36
    e. 40

1B-12. What is the scale of the diagram of the house?

Use the ruler provided to find, in terms of inches and feet, what scale has been used to construct the diagram.

1B-13. What would be the new length of the side of the house?

If you were to redraw the diagram using a scale of 3/4 inch = 10 feet, what would be the length of the side that is 48 feet?

    a. 3.0 in
    b. 3.6 in
    c. 5.6 in
    d. 7.5 in
    e. 12.0 in

1B-14. What is the ratio of sensors?

In a group of 1,200 student-built robots, there are 300 that use the touch sensor and the rest use the ultrasonic sensors. What is the ratio of robots that use the ultrasonic sensor to robots that use the touch sensor in the group?

    a. 4 to 3
    b. 4 to 1
    c. 3 to 1
    d. 1 to 4
    e. 1 to 3

1B-15. What's the circumference of the circle?
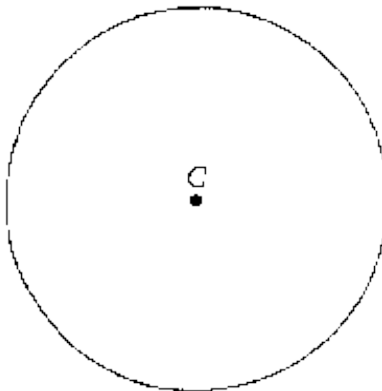
Using the centimeter ruler provided, find the circumference of the circle with center C above. (Use $\pi = 3.14$.)

_____ centimeters

1B-16. Where did the robot turn?
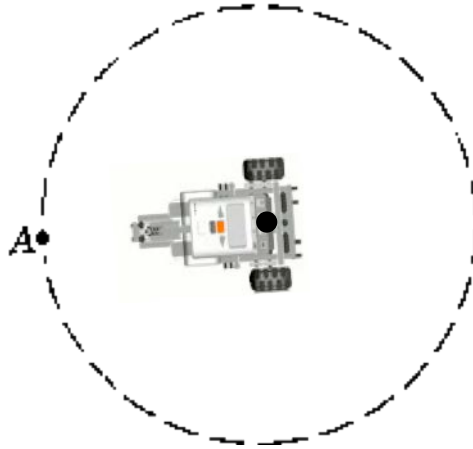
The robot is placed in the center of the circle shown, with the center of the circle at point C and the robot facing point A. Use the protractor to locate and label a point B that would illustrate the robot making a 235° point turn. Darken the outline of the circle where the robot would make its turn.

B.2   STUDY 2 – DESIGN BASED UNIT ASSESSMENT ITEMS

2-1.   What is n?

If $\dfrac{2}{25} = \dfrac{n}{500}$, then $n =$

    a. 10
    b. 20
    c. 30
    d. 40
    e. 50

2-2. Which enlargement?

Roxanne plans to enlarge her photograph, which is 4 inches by 6 inches. Which of the following enlargements maintains the same proportions as the original photograph? Justify your answer.

        5 inches by 7 inches   5 inches by 7½ inches

2-3. How tall is Mr. Tall in paper clips?
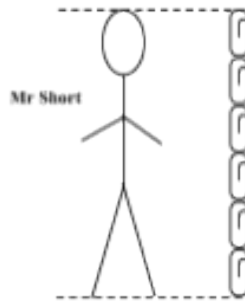
Mr Short

You can see the height of Mr. Short measured with paper clips. Mr. Short has a friend Mr. Tall. When we measure their heights with matchsticks:

    Mr. Short's height is four matchsticks.
    Mr. Tall's height is six matchsticks.

How many paper clips are needed for Mr. Tall's height?

2-4. How far away is the destination?

Car A and Car B are leaving the same place and going to the same destination. If it takes Car A 6 hours to get to the destination driving 25 miles per hour, and Car B 3 hours to get to the destination driving 50 miles per hour, how far away is the destination?

2-5. How many rulers?

Rulers cost $0.85 for two, including tax. How many rulers can Tom buy if he has $7.00?

2-6. Which car will arrive first?

Victor's van travels at a rate of 8 miles every 10 minutes. Sharon's sedan travels at a rate of 20 miles every 25 minutes.
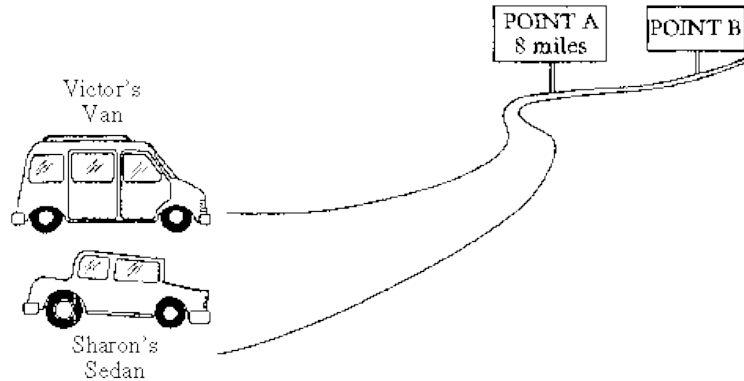
If both cars start at the same time, will Sharon's sedan reach point A, 8 miles away, before, at the same time, or after Victor's van? Explain your reasoning.

2-7. How many minutes?

A really fast runner runs for 40 minutes at 12 miles per hour. How many minutes would a much slower runner need to run in order to go the same distance if the slower runner keeps a pace of 8 miles per hour?

2-8. How many steps?

A giraffe moves forward 10 meters every step that she takes. A lion moves forward only 2 meters every step that she takes. If the giraffe takes 80 steps, how many steps must the lion take to cover the same distance as the giraffe?

2-9. What is the relationship of cards sold to profit?

Angela makes and sells special-occasion greeting cards. The table below shows the relationship between the number of cards sold and her profit. Based on the data in the table, which of the following equations shows how the number of cards sold and profit (in dollars) are related?

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Number Sold, n	4	0	5	2	3	6
Profit, p	$2.00	$0.00	$2.50	$1.00	$1.50	$3.00

a. $p = 2n$
b. $p = 0.5n$
c. $p = n - 2$
d. $p = 6 - n$
e. $p = n + 1$

2-10. What is the relationship of painters to time?

Justin runs a painting company. The table below shows the relationship between the number of painters he gets to work on a job and the number of days it takes to complete that job. Based on the data in the table, which of the following equations shows how the number of painters and the time to finish the job (in days) is related?

Number of Painters Working P	Time to Complete the Job T (days)
5	2
1	10
4	2.5
2	5

a. $T = 10 \times P$
b. $T = 7 - P$
c. $T = 2.5 \times P$
d. $T = 10/P$
e. $T = 3 + P$

B.3   STUDIES 3-6 – COMPETITION AND MODEL ELICITING ACTIVITY

ASSESSMENT ITEMS

3-1.   How many motor rotations has Alexa's robot done?

Alexa downloaded the same program to two identical robots. First, she starts one robot. A few moments later she starts the second robot. By the time the first robot had done 7 motor rotations, the second robot had done 3 motor rotations. How many total motor rotations will the first robot have completed by the time the second robot has completed 12 motor rotations?

3-2.   If you change the motor rotations, how far forward now?

A robot completes a move with 12 motor rotations and moves forward 14 centimeters. You modify the program to be 30 motor rotations. How far will it move forward now?

3-3.   How many for movement B?

Three different movements are programmed into a robot.

A: Move 15 *cm* straight forward ⟶
B: Move 10 *cm* straight forward ⟶
C: Move 5 *cm* straight forward ⟶

If it takes 2 motor rotations to do movement C, how many motor rotations are needed for movement B?

3-4.   How many rotations are needed?

A robot moved forward 6 centimeters when it was programmed to do 4 motor rotations. The programmer needed to make her robot move forward 24 centimeters. How many motor rotations does she need to enter in her program to do her move correctly?

3-5.   If you change the wheels, how far forward now?

A robot has a wheel circumference of 3 centimeters. The programmer successfully gets the robot to move forward 90 centimeters. The programmer then puts on new wheels with a wheel circumference of 7 centimeters and runs the same program. How far will the robot move forward now?

3-6.   Which robot moves further?

Robot A has wheels with a circumference of 3 centimeters and is programmed to do 3 motor rotations. Robot B has wheels with a circumference of 4 centimeters and is programmed to do 2 motor rotations. Which robot moves further?

  a.   Robot A moves further.
  b.   Robot B moves further.
  c.   They move the same distance.

3-7.   Which robot needs more motor rotations?

Robot A moves forward 10 centimeters in 4 motor rotations. Robot B moves forward 15 centimeters in 6 motor rotations. Which robot will need more motor rotations to move forward a distance of 40 centimeters?

  a.   Robot A will need more motor rotations.
  b.   Robot B will need more motor rotations.
  c.   They will both need the same number of motor rotations.

3-8.   Does Ed's rule work?

Ed got his robot working for one movement through trial-and-error. He got it to move straight forward 4 centimeters by programming it to do 10 motor rotations. Instead of doing trial-and-error again, he wanted to predict how many motor rotations he would need to put in his program to get his robot to go 6 centimeters. He said:

> *I know that when I want my robot to go further I need to add more motor rotations. Since 10 motor rotations gets me 4 centimeters, and 6 is two more than 4, I need to add two to the motor rotations also. 10 plus 2 is 12. That is why I think 12 motor rotations will work.*

Do you think Ed's idea works? If you do think his idea works, then explain why it makes sense. If you don't think his idea works, then explain why not and what he should do to fix his idea.

  a.   Yes, I do think Ed's idea works.
  b.   No, I don't think Ed's idea works.

3-9. Will one of Michelle's robots ever be twice as far as the other?

Michelle has two robots that she sets up side-by-side at the start line. She programmed them to move at the same motor speed straight forward and to keep moving until she pressed the stop button. One robot has a 3-centimeter wheel circumference and the other has a 5-centimeter wheel circumference. Will one robot ever be twice as far as the other robot from the start line? If so, which robot and when? If not, explain why not?

    a. Yes, one robot will eventually be twice as far as the other robot.
    b. No, one robot will never be twice as far as the other robot.

3-10. How tall is Mr. Tall in paper clips?
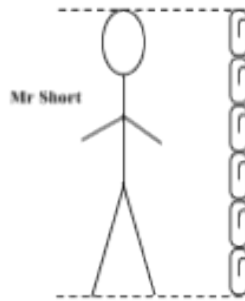
You can see the height of Mr. Short measured with paper clips. Mr. Short has a friend Mr. Tall. When we measure their heights with matchsticks:

Mr. Short's height is four matchsticks.
Mr. Tall's height is six matchsticks.

How many paper clips are needed for Mr. Tall's height?

3-11. How many dictionaries can it print? [Study 6 Only]

A printing press takes exactly 12 min to print 14 dictionaries. How many dictionaries can it print in 30 min?

APPENDIX C

INTERVIEW QUESTIONS

C.1 STUDY 1 – SCRIPTED INQUIRY REFLECTION INTERVIEW QUESTIONS

Introduction

I'd like to ask you a few questions so that I can understand your interests in robotics and mathematics and what makes you be motivated to try hard at something. Ultimately, we hope to better design classroom experiences, like this robotics unit, to take advantage of your interests and the things that motivate you, so the more honest you are the more we will be able to improve. None of your answer will affect your grade in the class.

Questions

1. *Tell me about what you thought of this course. Can you tell me about one time in this course when...*

   a. … you really tried (didn't try) hard? Why did you try hard?
   b. … you were really (un)interested? What about that time was (un)interesting?
   c. … you felt really frustrated (supported)? What about that time made you feel frustrated (supported)?
   d. Would you say that most of the time you were interested or uninterested?
   e. Would you say that most of the time you were felt frustrated or supported?
   f. What might motivate you to try harder more often in this course?

2. *Tell me about your thoughts on the relationship between math and robotics.*

   a. Is math helpful for doing robotics? Or is it really just unnecessary?
   b. Does math motivate you to try harder or less hard in robotics?
   c. Does robotics make it harder or easier to understand math?

3. *Tell me about what you see yourself doing after high school.*

   a. What made you choose the robotics/tech magnet at [school name]?
   b. Will you go to college? If so, what will you study?
   c. What kind of job do you plan on having?
   d. Will you be doing anything that uses mathematics and/or robotics?

4. *Tell me about one thing outside of class/school that you really care about and work hard in.*

   a. What is it that you like about it?
   b. What is it that motivates you to work hard at it?
   c. How is that different than the things you do in school?
   d. Are there things that you are motivated to work hard at even though you aren't interested in them?

General Follow-up Prompts

   Please tell me more about that…
   Is what you meant…?
   Is that true in general for you…?

C.2   STUDY 3 – COMPETITION DESIGN STRATEGY INTERVIEW QUESTIONS

Introduction

I'd like to ask you a few questions so that we can better understand what sort of teams are at this competition and the types of approaches that they took to solving the challenge. Ultimately, we hope to find out what are some of the things that makes a team successful. None of your answers will affect your performance in the competition. I'd like to start with a mentor (you) to ask about who your team is, and then I'd like to talk with a student to ask about your team's solution.

Questions (to Mentors)

1. *Number of Students (put a count next to each one)*

   a. # of 8th graders?
   b. # of 7th graders?
   c. # of 6th graders?
   d. # of 5th graders?
   e. # of 4th graders?
   f. # of 3rd graders?
   g. # of 2nd graders?
   h. Other? (describe)

2. *Competition Experience of Students (put a count next to each one)*

   a. # of Rookies?
   b. # w/ 1 Prior Competition?
   c. # w/ 2+ Prior Competitions?

3. *Number of Mentors/Coaches (put a count next to each one)*

   a. # of Professionals w/ robotics-related background?
   b. # of Teachers w/ robotics-related background?
   c. # of Professionals w/o robotics-related background?
   d. # of Teachers w/o robotics-related background?
   e. Other? (describe)

4. *Competition Experience of Mentors/Coaches (put a count next to each one)*

   a. # of Rookies?
   b. # w/ 1 Prior Competition?
   c. # w/ 2+ Prior Competitions?

5. *Hours Team has Met in Preparation (put a count next to each one)*

   a. # Prior to Last 2 Weeks?
   b. # in Last 2 Weeks?
   c. # in Total?

Questions (to Students)

6. *Programming Platform (choose all that apply)*

    a. NXT-G?
    b. ROBOTC?
    c. LabVIEW?
    d. easyC?

7. *Robot Platform (choose all that apply)*

    a. LEGO RCX?
    b. LEGO NXT?

8. *Robot Base Design (choose one)*

    a. Tankbot (RCX)?
    b. Robotics Educator (NXT)?
    c. Taskbot (NXT)?
    d. Domabot (NXT)?
    e. Completely original design?
    f. Other? (describe)
    g. Did you adapt it? (yes or no)
    h. If you did adapt it, how? (describe)

9. *Solution Strategy Straight – At any point in your solution, does your robot have to straight a specific distance? (yes or no)*

    a. What is the game context? (describe)
    b. Did you measure how far it has to go? (yes or no)
        i. If you did measure it, how far?
    c. What sensor did you use for that movement? (choose one)
        i. Touch?
        ii. Rotation?
        iii. Timer?
        iv. Sound?
        v. Light?
        vi. Ultrasonic?
    d. How did you determine the value for the sensor?
        i. N/A?
        ii. Unsystematic guess & test?
        iii. Systematic guess & test?
        iv. Proportion calculation?

     v. Overshoot?
    vi. Other? (describe)

10. *Solution Strategy Turns – At any point in your solution, does your robot have to turn a specific amount? (yes or no)*

  a. What is the game context? (describe)
  b. Did you measure how much it has to turn? (yes or no)
     i. If you did measure it, how much?
  c. What sensor did you use for that movement? (choose one)
     i. Touch?
    ii. Rotation?
    iii. Timer?
    iv. Sound?
     v. Light?
    vi. Ultrasonic?
  d. How did you determine the value for the sensor?
     i. N/A?
    ii. Unsystematic guess & test?
    iii. Systematic guess & test?
    iv. Proportion calculation?
     v. Overshoot?
    vi. Other? (describe)

11. *Solution Strategy Manipulators – At any point in your solution, do the manipulators on your robot have to move a specific amount? (yes or no)*

  a. What is the game context? (describe)
  b. Did you measure how much it they have to move? (yes or no)
     i. If you did measure it, how much?
  c. What sensor did you use for that movement? (choose one)
     i. Touch?
    ii. Rotation?
    iii. Timer?
    iv. Sound?
     v. Light?
    vi. Ultrasonic?
  d. How did you determine the value for the sensor?
     i. N/A?
    ii. Unsystematic guess & test?
    iii. Systematic guess & test?
    iv. Proportion calculation?
     v. Overshoot?
    vi. Other? (describe)

12. Other Strategies

     a. Thinking of the behaviors above, did you try a different way to determine the value for the sensor that didn't work? (yes or no)
          i. Yes? (describe)
     b. Did you ever try to use math for determining the value for the sensor? (yes or no)
          i. Yes? (describe)
     c. Did you ever use math in any other aspect of your work preparing for the competition? (yes or no)
          i. Yes? (describe)

C.3   STUDY 6 – MODEL ELICITING REFLECTION INTERVIEW QUESTIONS

Introduction

I'd like to ask you a few questions so we can better understand what you did this week and how the experience may have impacted your thinking about robots.

Questions

1. *How did you approach the problem of this game?*

     a. Did you use any strategies that we talked about this week?
     b. If so, where/when?
     c. If not, why not?
     d. Did you use any math at all?
     e. If so, where/when?
     f. If not, why not?
     g. If you had more time, how would you have approached it differently?

2. *How has this week influenced your thinking about robots?*

     a. How they work?
     b. How you understand them?
     c. How much you are interested in them?

3. *How has this week influenced your thinking about math?*

        a.   How you understand it?
        b.   How useful it is?
        c.   How much you are interested in it?

APPENDIX D

DESIGN BASED ENVIRONMENT ACTIVITY MATERIALS

Wksht 1 – The Design Task
Wksht 2 – Alternative Ideas
Wksht 3 – Dance Programming
Wksht 4 – Design Specification
Wksht 5 – Measurements Example
Wksht 6 – Measurements Instructions
Wksht 7 – First Synchronization Attempt
Wksht 8 – Evaluating the First Synchronization Attempt
Wksht 9 – Creating a Method to Synchronize the Robots
Wksht 10 – Data Table for Adjusting Straight Distances
Wksht 11 – Adjusting Straight Distances – Summary Table
Wksht 12 – Adjusting Straight Distances – Testing Your Strategy
Wksht 13 – Adjusting Straight Distances – Extending Your
Wksht 14 – Adjusting Straight Distances – Finalizing
Wksht 15 – Adjusting Straight Speed– Summary Table
Wksht 16 – Synchronizing Straight Moves – Distance and Speed
Wksht 17 – Adjusting Straight Speed – Finalizing

Robot Algebra

<div style="float:right;border:1px solid;">1</div>

THE DESIGN TASK – COORDINATING DANCING ROBOTS

You are about to begin a fun and creative project that will help you connect robotics, engineering, and mathematics. Your goal for the next several weeks will be to work together as a team to design and implement a dance routine for a group of LEGO robots. The challenging part will be coordinating each of the different robots so that they move in sync with each other.

Building Your Initial Dance Routine

You will eventually make your dance routine work on all of the robots in the class. However, so that every team has a robot to work with, you will start by planning your dance routine on just one robot that will be assigned to you. We will call this robot your *reference robot* (Robot 1). For your initial dance routine, you will be expected to choose a song and then build a series of robot moves that go along to the music.

Utilizing the Dance Floor Space

In order to encourage you to get your robot to move around a wide space, but not too wide of a space, the dance floor will be divided into 5 sections as shown in the figure below. Your team will receive a higher score on your dance routine the more sections your robot moves into at some point. 1 point will be awarded for each section your robot moves into, up to a maximum of 5 points. There is no requirement about where your robot starts or ends, so you can start your dance routine with your reference robot in any section and finish in any section. However, *if your robot stays within the outside boundaries of the dance floor, an additional 2 points will be awarded to your total score.*

202
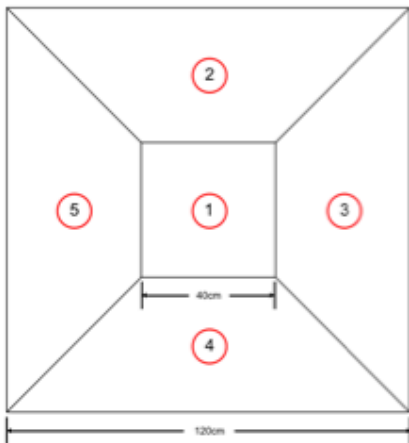
Robot Algebra

<div style="float:right; border:1px solid black;">1</div>

Incorporating the Range of Robot Movements

In order to encourage you to explore the full range of the robot's movements, your team will be awarded 1 point for your robot performing each of the following types of movements at any point in the routine, up to a maximum of 5 points for performing all of them. The movements available to you are:

1. Move forward straight
2. Move backward straight
3. Point turn (right or left)
4. Swing turn forward (right or left)
5. Swing turn backward (right or left)

Coordination to the Music

There will be a total of 9 points awarded for how well the robots' routine is coordinated to your song. Each of the following tasks can earn your routine 3 points.

1. The Robots' Dance must start with the music. It's a possibility if the music starts first before the robots start dancing but that gap should not be more than 3 seconds.

2. The Robots' Dance should ideally end with the song. If that proves to be too difficult to program then make sure the routine goes no more than 3 seconds longer or shorter than the song.

3. The dance routine should be connected and inspired by the music. This can be observed by listening to the song and looking to see if the Robots' movements are appropriate:

   A. A fast song would best support a routine that has lots of small quick movements. Think of the types of dances in America's Best Dance Crew or So You Think You can Dance. *But don't be limited by the song type - some classical music can be very fast (e.g. Flight of the Bumblebee).

   B. Slower songs should have less moves but each move should be longer. Think of people Waltzing or in Ballet. Again, don't be limited by your song genre (e.g. Outkast's Ms. Jackson or Celine Dion's My Heart Will Go On).

   The most important factor in choosing the dance or music is that they accurately represent your artistic vision.

Coordination between Robots

Eventually you will need to have all of the robots be coordinated with each other. But for now, we will focus just on designing your team's dance routine and getting your reference robot to perform the routine as you want it to be performed.

Robot Algebra

<div style="float:right;border:1px solid black;">1</div>

SONG LIST

You can choose from this library of songs to create your team's dance routine. The list includes choices from a variety of musical genres, and we encourage you to consider different alternatives.

SONG TITLE	ARTIST	TIME	GENRE
Back In Black	AC/DC	1:01	Rock
Bad	Michael Jackson	0:46	Pop
Beat It	Michael Jackson	0:55	Pop
Get Down On It	Kool and the Gang	0:44	R&B/Soul
Get Ready For This	2 Unlimited	0:33	Dance
Harder, Better, Faster, Stronger	Daft Punk	0:31	Electronic
In the Hall of the Mountain King	Edvard Grieg	0:41	Classical
Mama Said Knock You Out	LL Cool J	0:47	Rap
New Soul	Yael Naim	0:48	Pop
No Limit	2 Unlimited	0:30	Dance
Rhythm Nation	Janet Jackson	0:44	Pop
Seasons of Love	Broadway Musical Rent	0:55	Musical
September	Earth, Wind, & Fire	0:47	R&B/Soul
Take Five	Dave Brubeck Quartet	0:35	Jazz
The Imperial March	John Williams/ Star Wars	0:38	Classical
The Liberty Bell	John Phillips Sousa	0:44	Marching Band
Twilight Zone	2 Unlimited	0:30	Dance
We Will Rock You	Queen	0:36	Pop
Yeah!	Usher	0:35	R&B/Soul

Robot Algebra

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THE ROBOT DANCE TEAM

There are 4 different types of robots that we will work with. Each robot type has a unique letter assigned to it (A-D). In addition, there are two of each robot, a red one and a blue one. Each team will be assigned a different Robot 1 on which they will create their dance routine. Eventually all teams will get their dance routine to work on all 4 robots. The following is a list of the robots and which robot is assigned to each team:

TEAM	ROBOT 1 (RED)	ROBOT NAME	ROBOT COLOR
A	A	Ciara	Purple
B	B	Justin Timberlake	Green
C	C	Madonna	Silver
D	D	Michael Jackson	White

Ciara (Robot A)

PERSONALITY
Favorite Dance Genre: Salsa dancing **Favorite Color:** Purple **Favorite Dance Show:** So You Think You Can Dance
PHYSICAL STATS
Robot Base: GoBot Rover **Wheel Tread:** Smooth **Wheel Diameter:** 3.0 cm (small) **Wheel Base:** 11.0 cm (long) **Track Width:** 12.9 cm (medium)

A

Justin Timberlake (Robot B)

PERSONALITY
Favorite Dance Genre: Square dancing **Favorite Color:** Green **Favorite Dance Show:** Dancing with the Stars
PHYSICAL STATS
Robot Base: GoBot Rover **Wheel Tread:** Rough **Wheel Diameter:** 5.6 cm (medium) **Wheel Base:** 11.0 cm (long) **Track Width:** 13.7 cm (medium)

B

Robot Algebra

THE ROBOT DANCE TEAM (CONT.)

Madonna *(Robot C)*

PERSONALITY
Favorite Dance Genre: Swing dancing **Favorite Color:** Silver **Favorite Dance Show:** America's Best Dance Crew
PHYSICAL STATS
Robot Base: Domabot **Wheel Tread:** Smooth **Wheel Diameter:** 3.0 cm (small) **Wheel Base:** 8.0 cm (medium) **Track Width:** 16.5 cm (wide)

C

Michael Jackson *(Robot D)*

PERSONALITY
Favorite Dance Genre: Ballroom dancing **Favorite Color:** White **Favorite Dance Show:** Step It Up and Dance
PHYSICAL STATS
Robot Base: Domabot **Wheel Tread:** Rough **Wheel Diameter:** 5.6 cm (medium) **Wheel Base:** 8.0 cm (medium) **Track Width:** 17.1 cm (wide)

D

Robot Algebra   Name: _____ Date: _____ | 2 |

GENERATING ALTERNATIVE IDEAS

You should consider many different alternatives to your dance routine before deciding on one solution that your team will pursue. A functional specification is a general description of what you envision your dance routine to look like without working out all of the details. Consider at least two very different alternative designs for your team's dance routine. Write out a functional specification for those designs using the space provided below. For each functional specification, be sure to (a) designate what music you will coordinate with, (b) what your robots will do, and (c) how the movement of your robots fits with the music. Use the song list on the back for reference and attach additional pieces of paper if needed.

Dance Routine – Design #1

Dance Routine – Design #2

Dance Routine – Design #3 (optional)

Robot Algebra

DANCE PROGRAMMING

This document will help you get starting programming your first dance routine on the NXT robot using ROBOTC.

1. Get Your Assigned Robot

Each team will eventually make a routine that works on all of the robots. But to build your first routine, we will assign each team one robot to work with. Your team will work with the robot with the same letter as your team (e.g., Team A gets Robot A).

2. Open ROBOTC

We will be using RobotC to program the dance routines on the robot, so open "ROBOTC for Mindstorms 1.40" on your computer.

3. Open Sample Program

We have provided a sample dance program for you to get started. To open the program go to the "File" menu, choose "Open and Compile". Then navigate to "My Computer" > "TEAM? (F:)" > "DancingRobots". Finally, choose the file named "RA-Basic-Dance.c" and click "Open". When opened, the file should be similar to the one below.

RA-Basic-Dance.c

```
//////////////////////////////////////////////////////////////
// Comments
//////////////////////////////////////////////////////////////

#include "RobotAlgebra.c"

task main ()
{
    // Modify this line with your robot name
    initializeRobot(RMRobotEducator);

    startDanceRoutine();

    // Add your dance moves here
    moveForward(2, 30);
    turnPointRight(1, 30);
    turnSwingLeftBackward(1.5, 30);
    moveBackward(2, 30);

    endDanceRoutine();
}
```

Robot Algebra

<div style="float:right; border:1px solid; padding:4px;">3</div>

4. Rename the Program as Your Team's Dance

You can now modify this program so that the robot does what you tell it to do. First thing that you want to do is rename the program so that we know it's yours.

☐ Go to the "File" menu and choose "Save As…".

☐ Navigate to the "My Documents" folder and then open up the "RobotAlgebra" folder.

☐ Under "File name:" rename the program to be "TeamA-Dance.c", but use the team letter for your team. Your team letter is written on your robot.

5. Tell the Program to Use Your Robot

In order for your program to work correctly, you will need to modify the "initializeRobot" line in the program. Instead of "RMRobotEducator", you need to put in the initialization name of your robot. For example, if you are Team C and your robot is Madonna, then you would replace that line in the program with the following line:

```
initializeRobot(RMMadonna);
```

You can look up the initialization name for your robot in this table:

TEAM	ROBOT	NAME	INITIALIZATION
A	A	Ciara	RMCiara
B	B	Justin Timberlake	RMJustinTimberlake
C	C	Madonna	RMMadonna
D	D	Michael Jackson	RMMichaelJackson

6. Download the Dance Program from the Computer to Your Robot

To download this program to the robot and watch it run, do the following:

☐ Connect the robot to the computer using the USB cable and turn the robot on by pressing the orange square button on the NXT brick.

☐ Go to the "Robot" menu, and then select "Compile and Download Program" or "Download Program" (whichever choice is available).

☐ Several new windows may appear. If they do, close them before moving on.

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7. Run the Dance Program on Your Robot

Now you can run the dance program.

☐ Disconnect the robot from the USB cable and place it on the ground.

☐ Navigate to the program by pressing the orange square button on the NXT brick for "My Files". Press it again for "Software Files". Press it again for "TeamA-Dance". And then one last time, to "Run" the program.

☐ The robot may take a few seconds to get started.

☐ It will then tell you that it is waiting for the touch sensor to be pushed. Push the touch sensor and watch what happens.

8. Modify, Save and Download Changes

You can now modify this program so that the robot does what you tell it to do for your dance routine. You can delete the dance moves that are already in the program, or keep them and add your own. You can add as many moves as you want, but it is a good idea to keep the routine simple to start. You can always add more later. Every time you make a change, save the program, and then download it again to your robot to see the change in the routine.

9. Make Your Dance Routine

Below is a list of the moves that are available to you to program your robot. For each move, you can fill in a particular value for how much to rotate the wheels:

Available Move Commands

```
// Straight Moves
moveForward (revolutionsToRotateWheels, motorPower);
moveBackward (revolutionsToRotateWheels, motorPower);
// Point Turns
turnPointRight (revolutionsToRotateWheels, motorPower);
turnPointLeft (revolutionsToRotateWheels, motorPower);
// Swing Turns
turnSwingRightForward (revolutionsToRotateLeftWheel, motorPower);
turnSwingRightBackward (revolutionsToRotateRightWheel, motorPower);
turnSwingLeftForward (revolutionsToRotateRightWheel, motorPower);
turnSwingLeftBackward (revolutionsToRotateLeftWheel, motorPower);
```

More information on the commands that you can use in your program is provided on the next pages. But don't be afraid to try things out and see what happens. Good luck. =)

Robot Algebra

<div style="border:1px solid;display:inline-block">3</div>

ROBOT MOVEMENTS

When working with your robots in this unit, the robot will only be capable of performing a limited set of movements. The following is a list of the movements that your robot is capable of performing, along with a short description of how the robot does the move and the move command. The thin arrows and marks indicate the movement of the wheels.

Move Forward

Both wheels rotate forward at the same speed and the robot moves straight forward.

```
moveForward (revolutionsToRotateWheels, motorPower);
```

Move Backward

Both wheels rotate forward at the same speed and the robot moves straight backward.

```
moveBackward (revolutionsToRotateWheels, motorPower);
```

Point Turn Right

Both wheels rotate at the same speed, but the left wheel rotates forward and the right wheel rotates backward, so the robot turns to its right around its center. This makes a sharp turn in place.

```
turnPointRight (revolutionsToRotateWheels, motorPower);
```

Point Turn Left

Both wheels rotate at the same speed, but the right wheel rotates forward and the left wheel rotates backward, so the robot turns to its left around its center. This makes a sharp turn in place.

```
turnPointLeft (revolutionsToRotateWheels, motorPower);
```

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Swing Turn Forward Right

The left wheel rotates forward and the right wheel does not move, so the robot pivots around the right wheel as it turns forward. This makes a wider turn.

```
turnSwingRightForward (revolutionsToRotateLeftWheel, motorPower);
```

Swing Turn Backward Right

The right wheel rotates backward and the left wheel does not move, so the robot pivots around the left wheel as it turns backward. This makes a wider turn.

```
turnSwingRightBackward (revolutionsToRotateRightWheel, motorPower);
```

Swing Turn Forward Left

The right wheel rotates forward and the left wheel does not move, so the robot pivots around the left wheel as it turns forward. This makes a wider turn.

```
turnSwingLeftForward (revolutionsToRotateRightWheel, motorPower);
```

Swing Turn Backward Left

The left wheel rotates backward and the right wheel does not move, so the robo pivots around the left wheel as it turns backward. This makes a wider turn.

```
turnSwingLeftBackward (revolutionsToRotateLeftWheel, motorPower);
```

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OTHER USEFUL PROGRAMMING COMMANDS

There are a number of other useful commands that you can use in your dance program. The following is a list of those commands:

Initialize Robot

This command prepares your robot before starting a dance routine. It tells the program what robot is doing the dancing. Therefore, it should be changed whenever you put your dance program on a different type of robot. The following 4 values can be used for the robot model: RMCiara, RMJustinTimberlake, RMMadonna, RMMichaelJackson. You should use the model that corresponds to the robot you are using.

```
initializeRobot (robotModel);
```

Start Dance Routine

This command should be called right before you are about to start your first dance move. This command will indicate that the routine is about to start, and will wait for your starting command. There are 4 options for a starting command: (1) press the touch sensor (WFTouch), this is the default; (2) make a loud noise (WFLoud); (3) wait for a set number of seconds (WFTime); or (4) just move right into the routine (WFNothing).

```
startDanceRoutine (waitForRoutine, waitForParam);
```

End Dance Routine

This commands indicates the dance routine is over and indicates that on the robot screen.

```
endDanceRoutine ();
```

Use Step Through Mode

This command helps you to analyze or measure each different dance move in your dance routine. When it is included in your program, you robot will make only one move at a time, then will pause an wait for you to hit the touch sensor before moving onto the next move.

```
useStepThroughMode ();
```

Robot Algebra   Team: _____ Date: _____   4

DESIGN SPECIFICATION

Our Team's Name:

Song for Our Dance Routine:

Functional Description of Our Dance Routine Design:

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DESIGN SPECIFICATION (CONT.)

Dance Move List
(List the sequence of moves made by your robot.)

Robot 1 Name: _____

MOVE #	ROBOT MOVEMENT	MOTOR ROTATIONS (REV)	MOTOR SPEED (REV/SEC)	DISTANCE TRAVELED (CM)	ANGLE TURNED (°)	TIME TAKEN (SEC)	SONG NOTES
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							

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DESIGN SPECIFICATION (CONT.)

Dance Move List (cont.)
(List the sequence of moves made by your robot.)

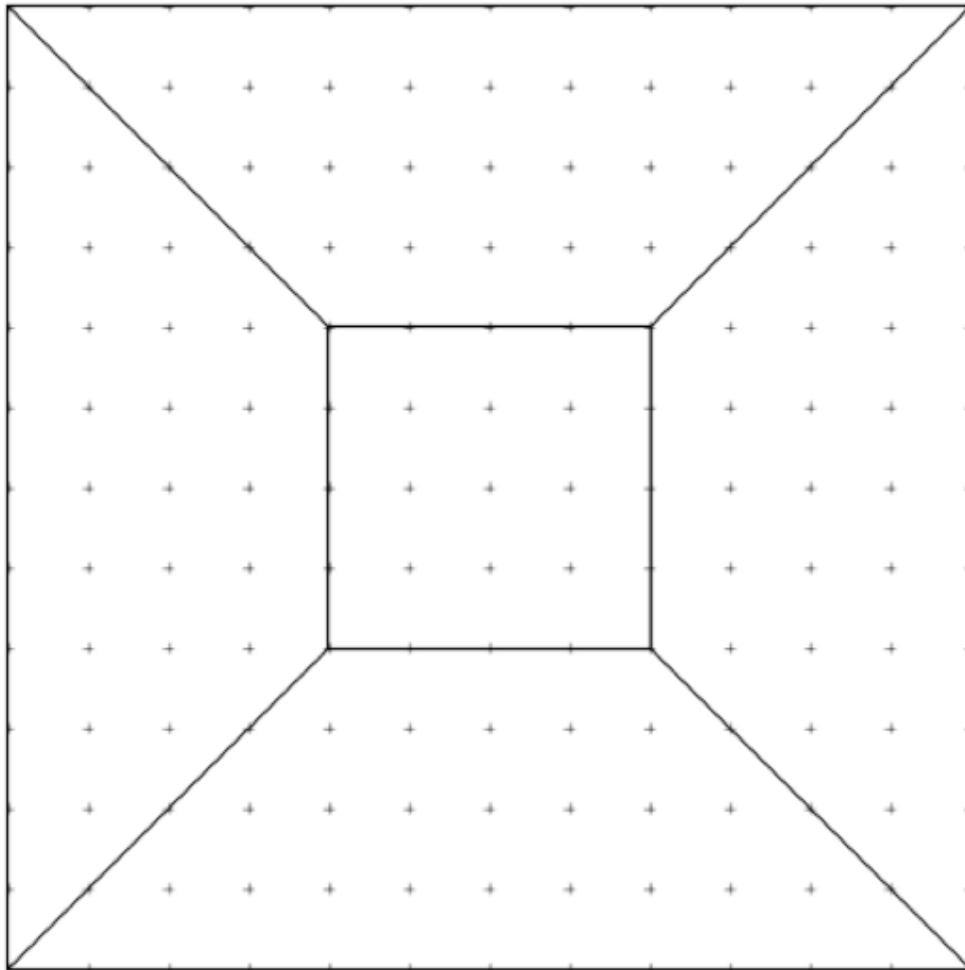| MOVE #	ROBOT MOVEMENT	MOTOR ROTATIONS (REV)	MOTOR SPEED (REV/SEC)	DISTANCE TRAVELED (CM)	ANGLE TURNED (°)	TIME TAKEN (SEC)	SONG NOTES
12							
13							
14							
15							
16							
17							
18							
19							
20							
21							
22							

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Robot Algebra

DESIGN SPECIFICATION (CONT.)

Blueprint Diagram
(Map out the moves made by your robot.)

Scale
Diagram – 1.5cm :: Real World – 10cm

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DESIGN SPECIFICATION – HOW TO DO MEASUREMENTS

These instructions will help you with the part of your design specification that asks you to measure each move in your dance routine. Measuring each move will provide you with clear targets so that it will be easier to teach the dance routine to all of the different robots.

1. **Save As a New Program for Measuring** – Open your dance program, save it as a new name that indicates that this program is for measuring (e.g., "TeamZ-Measure.c").

2. **Write Down Program Information** – This includes the type of movement (e.g., Move Forward, Turn Point Left, etc.), the number of motor rotations, and the motor speed.

3. **Turn on Step Through Mode** – This sets up your robot so that it does one move then waits for you to press the touch sensor before moving on to the next move.

4. **Comment Out Moves You are NOT Focused On** – You can use a single line comment ("// commented text here") or a multiline comment ("/*commented text here */").

5. **Compile and Download the Program** – Use the menu in ROBOTC to compile the program and download it to the robot.

6. **Mark the Starting Position of the Wheel** – Place the robot on a sheet of paper and trace one of the wheels.

7. **Run a Move on the Robot** – Find the program on the robot and run it.

8. **Mark the Ending Position of the Wheel** – Trace the same wheel as you traced when marking the starting position.

9. **Measure from the Start to the End Position** –

    a. **For Straight Moves** – Measure from the back of the trace of starting position of the wheel to the back of the trace of the ending position of the wheel.

    b. **For Turning Moves** – Extend the traced lines of the wheel so that they meet at an angle. Measure that angle using a protractor.

10. **Measure Multiple Times** – This helps you to make sure you have a value that is representative of what the robot does the majority of the time.

11. **Record the Time** – This is timed by the robot's internal clock and is presented on the screen as soon as the move completes.

12. **Move to the Next Move** – Measure at least three moves of each type (straights, turning). Eventually you will have to do them all in order to synchronize your dance routine across all the robots, so the more you do now the better.

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DESIGN SPECIFICATION – MEASUREMENT EXAMPLE

Dance Move List
(List the sequence of moves made by your robot.)

Robot 1 Name: _____ Justin Timberlake (Robot B) _____

MOVE #	ROBOT MOVEMENT	MOTOR ROTATIONS (REV)	MOTOR SPEED (REV/SEC)	DISTANCE TRAVELED (CM)	ANGLE TURNED (°)	TIME TAKEN (SEC)	SONG NOTES
1	Move Forward	3.00 r	0.60 r/s				Intro – slow walk forward
2	Turn Point Left	0.61 r	0.80 r/s				Chorus begins
3	Turn Point Right	0.31 r	0.80 r/s				
4	Move Backward	1.60 r	0.50 r/s				
5	Turn Point Right	0.92 r	0.80 r/s				
6	Turn Point Left	0.31 r	0.80 r/s				
7	Move Backward	1.50 r	0.50 r/s				
8	Turn Point Left	0.92 r	0.80 r/s				
9	Turn Point Right	0.31 r	0.80 r/s				
10	Move Forward	1.50 r	0.50 r/s				

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DESIGN SPECIFICATION – HOW TO DO MEASUREMENTS

These instructions will help you with the part of your design specification that asks you to measure each move in your dance routine. Measuring each move will provide you with clear targets so that it will be easier to teach the dance routine to all of the different robots.

Measuring Straight Moves

1. **Mark the Starting Position of the Wheel** – Place the robot on a sheet of paper and trace one of the wheels. Add an arrow to the trace to indicate which way the robot is facing. Mark the trace with an "S" to indicate it is the STARTING position.

2. **Run a Move on the Robot and Mark the Ending Position of the Wheel** – Find the program on the robot and run it. Then trace the same wheel as you traced when marking the starting position. Add an arrow to the trace to indicate which way the robot is facing. Mark the trace with an "E" to indicate it is the ENDING position.

3. **Measure from the Start to the End Position** – Remove the robot and measure the change from the start to the end position. Measure from the back of the trace of starting position of the wheel to the back of the trace of the ending position of the wheel. Since this movement is straight, use a straight ruler or meter stick to measure the distance traveled. Record the value in centimeters.

Step 1 Mark the Start	Step 2 Run the Move and Mark the End	Step 3 Measure the Distance Traveled

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DESIGN SPECIFICATION – HOW TO DO MEASUREMENTS (CONT.)

Measuring Turning Moves

1. **Mark the Starting Position of the Wheel** – Place the robot on a sheet of paper and trace one of the wheels. Add an arrow to the trace to indicate which way the robot is facing. Mark the trace with an "S" to indicate it is the STARTING position.

2. **Run a Move on the Robot and Mark the Ending Position of the Wheel** – Find the program on the robot and run it. Then trace the same wheel as you traced when marking the starting position. Add an arrow to the trace to indicate which way the robot is facing. Mark the trace with an "E" to indicate it is the ENDING position.

3. **Measure from the Start to the End Position** – Remove the robot and measure the change from the start to the end position. Extend the traced lines of the wheel so that they meet at an angle. Since this movement is a turn, use a protractor to measure the angle turned from the start direction to the end direction. Record the value in degrees.

    a. When using the protractor, always place the center of the protractor where the two extended lines meet.

    b. Put the bottom of the protractor (the zero degree mark) on the extended start line.

    c. Read the value where the extended end line meets the protractor.

    d. There are always two values on the protractor. *Use the value that makes the most sense.* For example, if you know the turn was less than 90°, then the angle turned must be less than 90°. Similarly, if your robot made a long turn, you may have to add 180°, 360° or more to the measured value to get the actual angle turned.
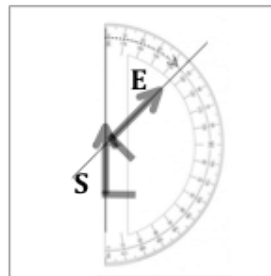
Step 1	Step 2	Step 3
Mark the Start	Run the Move and Mark the End	Measure the Angle Turned

Robot Algebra   Team: _____ Date: _____ | 7 |

FIRST SYNCHRONIZATION ATTEMPT

The overall goal of this project is to get your dance routine to work on *all of the robots* in our robot dance team even though you designed your dance routine on only one of them. We are now going to take the next step to figuring out how we can make all the other robots in the dance team perform the same dance moves in sync with each other. We are going to put your original dance routine program on a second robot and just see what happens.

Putting the Original Dance Program on a Second Robot

Each team will be assigned a second robot that they will use to make their first synchronization attempt. The column Robot 2 in the following table lists the second robot that each team will use. For example, Team B will use Robot C (Madonna) as its second robot and Team D will use Robot A (Ciara).

TEAM	ROBOT 2 (BLUE)	ROBOT 2 NAME	ROBOT 2 PROGRAM	ROBOT 2 INITIALIZATION NAME
A	D	Michael Jackson	TeamA-RD-Sync1.c	RMMichaelJackson
B	C	Madonna	TeamB-RC-Sync1.c	RMMadonna
C	B	Justin Timberlake	TeamC-RB-Sync1.c	RMJustinTimberlake
D	A	Ciara	TeamD-RA-Sync1.c	RMCiara

1. Open your original dance program in ROBOTC (e.g., "TeamC-RC-Dance.c") and download this program to your original robot (e.g., "Robot C").

2. Save the file as a new name to indicate that you are putting this program on a second robot to make a first attempt at synchronization. The table above tells you what to name your program. For example, if you are Team A, then save the new program as "TeamA-RD-Sync1.c".

3. Modify only one line in the program, the line that tells the program which robot you are using. Change that line so that it has the name of the new robot you will be using.

```
initializeRobot(RMMadonna);
```

4. Get your second robot and download this new program to that robot.

5. Talk with your team about what you think will happen. Then run your first robot and this second robot side-by-side and see what happens. Your instructor will videotape this so you can view it again later.

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FIRST SYNCHRONIZATION ATTEMPT (CONT.)

Program Naming and Organization

Now that we starting to use many different robots it is going to be harder to keep all of your programs organized. In order to make sure you don't lose any of your hard work, but still give you the freedom to make changes and try new things, we are going to implement a way to name all the different programs. This is called a naming scheme. The naming scheme will help us to stay organized.

The naming scheme will have 3 parts:

1. The letter of the team that created the program (e.g., "TeamA", "TeamD", etc.).

2. The letter of the robot that the program was created for (e.g., "RB" for Robot B).

3. A one-word description of the program (e.g., "Measure" for measuring the dance moves in the original dance routine, "Sync1" for the first synchronization attempt, "Sync2" for the second synchronization attempt, etc.).

Here is an example of the filenames that Team Z (Eli's team) would use:

PROGRAM NAME	DESCRIPTION
TeamZ-RZ-Dance.c	The original dance routine design by *Team Z* using *Robot Z*.
TeamZ-RZ-Measure.c	The program used to measure the distance traveled and angle turned for all of the dance moves for *Team Z* using *Robot Z*.
TeamZ-RC-Sync1.c	The program used by *Team Z* as a first attempt at syncing the original dance routine onto a second robot, *Robot C*.
TeamZ-RB-Sync1.c	The program used by *Team Z* as a first attempt at syncing the original dance routine onto a third robot, *Robot B*.
TeamZ-RC-Sync2.c	The program used by *Team Z* as a second attempt at syncing the original dance routine onto a second robot, *Robot C*.

Make sure that all of your existing programs and all future programs follow this naming scheme so that you can say organized.

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EVALUATING THE FIRST SYNCHRONIZATION ATTEMPT

Our ultimate design goal:

> **To get *many, different* robots to do a *synchronized* dance.**

"Many, different robots" means that the robots are going to come in all different shapes and sizes. *We are not going to change the robots.* We have to work with what we are given. Instead, we are going to program them so that they are synchronized with each other.

1. What does it mean to be *synchronized*?

    Your Ideas:

    Other Ideas from the Class:

2. Evaluating the robots side-by-side.

	DANCING DIFFERENCES? (OUT OF SYNC)	POSSIBLE REASONS FOR DIFFERENCES
Your Ideas		
Other Ideas from the Class		

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3. What do we need to know to solve the problem (get the robots to be in sync)?

List all the different aspects of the robots and/or the programming of the robots that you feel you need to know more about in order to get all the robots to do your dance routine synchronized with each other.

WHAT DO WE NEED TO KNOW TO SOLVE THE PROBLEM?
Your Ideas
Other Ideas from the Class

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CREATING A METHOD TO SYNCHRONIZE THE ROBOTS

A key learning question developed by the class last session:

How **fast** do you have to program one robot to be in sync with the other one?

You have control over the *robot program*, but you want to modify the *behavior of the robot in the real world*. So how does the **robot program** control the **robot behavior**?

```
moveForward(3.0, 0.8); // moveForward( Motor Rotations, Motor Speed )
```

* For this example move, Robot D traveled a distance of 58 centimeters in 3.73 seconds (its behavior).

1. If we want to control how *fast* the robot moves in the real world (its behavior), what should we change in the program?

   Your Ideas:

   Other Ideas from the Class:

Work in your group and create a method that you can use to adjust the second robot so that it goes just as fast as the first robot. On the back of this page, describe your method as a series of steps (e.g., "Step 1 – Determine the speed of Robot 1; call that the *Target Speed*"). You method must meet the following conditions:

   a. You may only adjust the program for the second robot (the program for the first robot shouldn't be changed).
   b. You may only change one of the parameters in the program (either motor rotations or motor speed, but not both).
   c. You must include quantitative measurements that determine when Robot 2 is just as fast as Robot 1 and how far off they are at every attempt (you can't just look with your eyes).
      Hint: Use your measurements from the Design Specification document (#4).
   d. You must have a way to record every attempt, even the ones that don't work, so you can evaluate your progress.
   e. You should only work on one movement at a time (don't fix the whole program at once).
      Hint: Use the example move from Robot D above as an example.

Robot Algebra

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CREATING A METHOD TO SYNCHRONIZE THE ROBOTS (CONT.)

Describe your team's procedure in the space below:

Robot Algebra

Team: _____ Date: _____

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DATA TABLE FOR ADJUSTING STRAIGHT DISTANCES

Note: You will only be using one robot. Robot 1 is just for reference. Robot 2 is the one you will be programming and adjusting as you attempt to make it synchronized with Robot 1.

Step 1: Choose a straight move from your dance program.

Step 2: Write down the move number from your dance program.

Step 3: Explain your current strategy for deciding what value for the motor rotations to use for this trial.

Step 4: Write down the target distance.

Step 5: Run that movement on Robot 2 and measure the actual distance it moves forward.

Step 6: In the evaluation column, indicate the success of your current strategy, and write any notes about how well it worked.

a. Got it – our strategy worked
b. Getting close – strategy is working, next try should do it
c. On our way – making progress, keep our current strategy
d. Really far off – we need to get a whole new strategy
e. Ugh – we have no clue what we are doing

Step 7: Do more trials using different values for the motor rotations until the target distance and actual distance are equal.

Robot 1 Name: _____ Robot 2 Name: _____

MOVE #	TRIAL #	STRATEGY FOR THIS TRIAL	MOTOR ROTATIONS (ROT)	TARGET ROBOT DISTANCE (CM)	ACTUAL ROBOT DISTANCE (CM)	EVALUATION
	1	As a first attempt, we took the motor rotations directly from the Robot 1 dance program				A B C D E
2						A B C D E

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WKSHT 10 – DATA TABLE FOR ADJUSTING STRAIGHT DISTANCES
(PAGE 2 OF 2)

MOVE #	TRIAL #	STRATEGY FOR THIS TRIAL	MOTOR ROTATIONS (ROT)	TARGET ROBOT DISTANCE (CM)	ACTUAL ROBOT DISTANCE (CM)	EVALUATION
						A B C D E
						A B C D E
						A B C D E
						A B C D E
						A B C D E
						A B C D E

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Robot Algebra   Team: _____ Date: _____   11

ADJUSTED STRAIGHT DISTANCES – SUMMARY TABLE

Use this sheet to summarize how your team adjusted the straight distances on your second robot to sync with your original robot. Whenever you figure out the correct adjustment for the motor rotations of Robot 2, write down the motor rotations needed in the table below.

Robot 2 Name: _____

MOVE #	ROBOT MOVEMENT	DISTANCE TRAVELED (CM)	ROBOT 2 MOTOR ROTATIONS (REV)	NUMBER OF TRIALS
-	Move Forward	40 cm		
-	Move Forward	60 cm		

If your dance routine doesn't have at least 5 different straight movements, then make up distances that you want the robot to go and adjust the motor rotations until the Robot 2 goes those distances. After getting the distance adjusted for 7 straight movements, answer the questions on the back of this sheet.

Robot Algebra

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ADJUSTED STRAIGHT DISTANCES – REFLECTION

1. What was the **best** strategy your team used for adjusting the motor rotations? Explain why that strategy worked better than the other strategies your team tried.

2. Without actually running your robots, determine how many motor rotations you would need to put in the program to make each robot travel a straight distance of 10,000cm.

3. Write a description that could be used to compute the number of motor rotations needed to program your Robot 2 to travel any given straight distance.
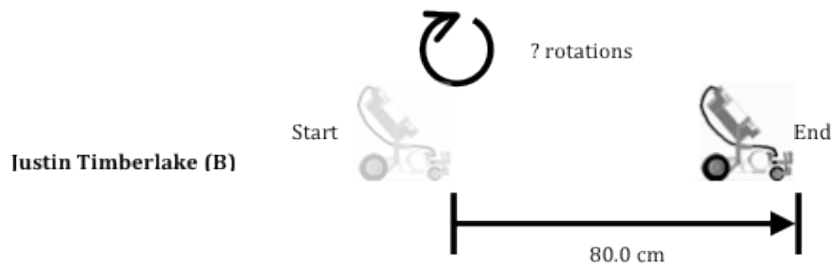
Robot Algebra

Team: _____ Date: _____

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ADJUSTING STRAIGHT DISTANCES – TESTING YOUR STRATEGY

Given the following problem, use your team's best strategy for adjusting straight distances to figure out how many motor rotations will make Robot 2 go the same distance as Robot 1.

Robot 1 Name: _____**Madonna (C)**_____   Robot 2 Name: _____**Justin Timberlake (B)**_____

I made a dance move for my routine that programs Madonna (Robot C) to move forward 8.50 motor rotations. When I measure that move, Madonna moves forward 80.0 cm. Now I want to get my second robot, Justin Timberlake (Robot B), to go the same distance so the two robots are synchronized. How many motor rotations do I need to put into Justin Timberlake's program to get it to also move forward 80.0 cm?

8.50 rotations

Start   End

Madonna (C)

80.0 cm

? rotations

Start   End

Justin Timberlake (B)

80.0 cm
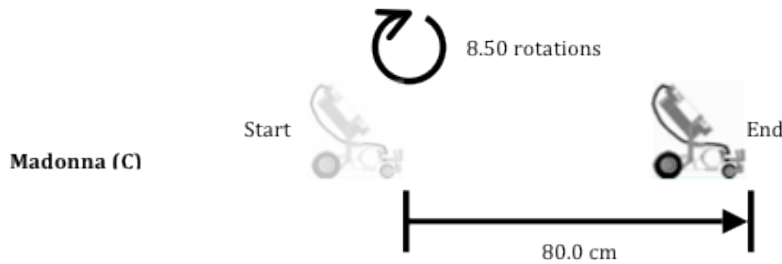
Robot Algebra   Team: _____ Date: _____   13

ADJUSTING STRAIGHT DISTANCES – EXTENDING YOUR STRATEGY

Use the strategy that makes the most sense to you to figure out how many motor rotations will make Robot 2 go the distance specified. Show your work in the space below.

Robot 1 Name: ___**Madonna (C)**___   Robot 2 Name: ___**Justin Timberlake (B)**___

Problem 1

Now I want to get my second robot, Justin Timberlake (Robot B), to go 200 cm. How many motor rotations do I need to put into Justin Timberlake's program to get it to move forward 200.0 cm?

? rotations

Start   End

Justin Timberlake (B)

200.0 cm

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Robot Algebra

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Problem 2

Now I want to get my second robot, Justin Timberlake (Robot B), to go 10,000 cm. How many motor rotations do I need to put into Justin Timberlake's program to get it to move forward 10,000.0 cm?

? rotations

Start   End

Justin Timberlake (B)

10,000.0 cm

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Robot Algebra   Team: _____ Date: _____   14

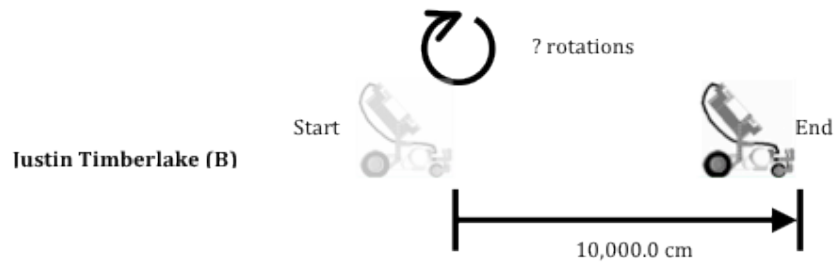## ADJUSTING STRAIGHT DISTANCES – FINALIZING

Today, your team will finish adjusting straight distances on your Robot 1 and Robot 2 and will document your progress in a YouTube video.

Setting Up Your Robots

Make two dance programs, one program for each robot, that include your entire dance routine. Save them with a filename that includes your team letter, the letter of the robot, and "Sync5". For example, "Team**A**-R**D**-Sync**5**.c". Make sure to adjust the motor rotations for the second robot so it goes the same distance as your first robot. Feel free to use whatever solution you want to do this.

Team A's Solution for Adjusting Straight Distances

1. We would find out the number of centimeters the robot moves forward when programmed to go 1 motor rotation.

2. We divide the total distance we want the robot to go by the number we found in Step 1 (how far the robot goes in 1 rotation). This gives us the number of rotations we would put in our program.

Your YouTube Progress Video

Every team will run their robots side-by-side with their music after adjusting the straight distances. We will interview you in the video to find out what you have done. Here are the questions we will ask:

Before Running the Robots

1. Last time we saw your robots dancing you had the same program on both robots and they were way out of sync. What have you changed in the program since last time? Why?

2. What should I expect to see different from last time in terms of the robots' dancing being in sync?

After Running the Robots

3. Did the robots do what you thought they would do?

4. Are they in sync? If not, what do you think the problem is?

5. What do you need to do now to get them more in sync?

Robot Algebra   Team: _____**Team A**_____ Date: _____   15

ADJUSTED STRAIGHT SPEED – SUMMARY TABLE

Use this sheet to summarize how your team adjusted the straight speed on your second robot to sync with your original robot. Whenever you figure out the correct adjustment for Robot 2, write down the values needed in the table below.

Robot 2 Name: _____**Michael Jackson (Robot D)**_____

MOVE #	ROBOT MOVEMENT	DISTANCE TRAVELED (CM)	TIME TAKEN (SEC)	ROBOT 2 MOTOR ROTATIONS (REV)	ROBOT 2 MOTOR SPEED (REV / SEC)	VALUES YOU TRIED THAT DIDN'T WORK
5	Move Backward	32 cm	1.98 sec			
7	Move Forward	26 cm	1.22 sec			
10	Move Backward	39 cm	1.97 sec			
14	Move Backward	27 cm	2.98 sec			
15	Move Forward	10 cm	0.50 sec			
16	Move Backward	30 cm	2.97 sec			
-	Move Forward	40 cm	5.00 sec			
-	Move Forward	60 cm	5.00 sec			

After getting the speed adjusted for the straight movements, answer the questions on the back of this sheet.

LRDC & NREC   Page 1 of 2

Robot Algebra

ADJUSTED STRAIGHT DISTANCES – REFLECTION

1. What was the **best** strategy your team used for adjusting to synchronize speed? Explain why that strategy worked better than the other strategies your team tried.

2. Without actually running your robots, determine how to set the motor rotations and motor speed in the program to make your Robot 2 travel a straight distance of 10,000cm in 700 seconds.

3. Write a description that could be used to compute the motor rotations and the motor speed needed to program your Robot 2 to travel any given straight distance in any given amount of time.

Robot Algebra

Team: _____ Date: _____

10

DATA TABLE FOR ADJUSTING STRAIGHT SPEED

Note: You will only be using one robot. Robot 1 is just for reference. Robot 2 is the one you will be programming and adjusting as you attempt to make it synchronized with Robot 1.

Step 1: Choose a straight move from your dance program.

Step 2: Write down the move number from your dance program.

Step 3: Explain your current strategy for deciding what value for the motor speed to use for this trial.

Step 4: Write down the target robot speed.

Step 5: Run that movement on Robot 2 and find the actual robot speed the robot moves forward.

Step 6: In the evaluation column, indicate the success of your current strategy, and write any notes about how well it worked.

   a. Got it – our strategy worked
   b. Getting close – strategy is working, next try should do it
   c. On our way – making progress, keep our current strategy
   d. Really far off – we need to get a whole new strategy
   e. Ugh – we have no clue what we are doing

Step 7: Do more trials using different values for the motor speed until the target robot speed and actual robot speed are equal.

Robot 1 Name: _____

Robot 2 Name: _____

MOVE #	TRIAL #	STRATEGY FOR THIS TRIAL.	MOTOR SPEED (ROT./SEC)	TARGET ROBOT SPEED (CM/SEC)	ACTUAL ROBOT SPEED (CM/SEC)	EVALUATION
	1	As a first attempt, we took the motor speed directly from the Robot 1 dance program				A B C D E
2						A B C D E

LRDC & NREC

Page 1 of 4

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MOVE #	TRIAL #	STRATEGY FOR THIS TRIAL	MOTOR SPEED (ROT/SEC)	TARGET ROBOT SPEED (CM/SEC)	ACTUAL ROBOT SPEED (CM/SEC)	EVALUATION
						A B C D E
						A B C D E
						A B C D E
						A B C D E
						A B C D E
						A B C D E

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Robot Algebra   Team: _____ Date: _____   16

SYNCHRONIZING STRAIGHT MOVES – DISTANCE AND SPEED RECAP

Synchronizing Distance

When thinking about distance, what we know is how far we want the robot to go (the total/target distance), and what we need to know is how many times to rotate the wheels so we can put that value in our program.

   Total/Target Distance – 180 cm   →   Motor Rotations – ?? rot

Team A had an excellent strategy. They knew that how far a robot moved forward each wheel rotation depended on the size of its wheel. So if you knew how far a robot moved forward in 1 wheel rotation, then you could use that information to figure out how many times to rotate the wheel for any distance by dividing. Let's explore their strategy more.

Let's call the amount a robot moves forward in one wheel rotation its **1rotation-distance**. Now we want to figure out how many **1rotation-distances** are needed to go the **total-distance**. This is just like dividing, as we are asking how many parts (1rotation-distances) are needed to make up the whole (total-distance).

$$\frac{\textbf{total-distance}}{\textbf{1rotation-distance}} \; = \; \textbf{motor-rotations}$$

Example 1

For example, Madonna's (Robot C) 1rotation-distance is 9.4 cm. Let's say our dance move needs our robots to move forward a total-distance of 65 cm. How many of Madonna's 1rotation-distances are needed to move forward the total-distance?
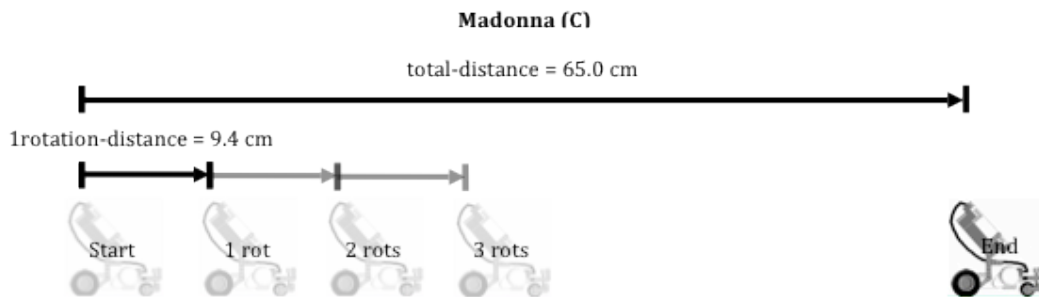
Madonna (C)

total-distance = 65.0 cm

1rotation-distance = 9.4 cm

Start   1 rot   2 rots   3 rots   End

Robot Algebra

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Synchronizing Speed

As many of you have already tried, we can think of synchronizing speed in a similar way as synchronizing distance. For this part, we know how far we want the robot to go forward for a dance move and we know how much time we have to do that dance move. What we want to know is how fast to rotate the motors, in units of rotations per second.

This problem is almost easier, because in addition to knowing how far we want every robot to go, we also figured out how many motor rotations are needed for each robot when we synchronized the distance.

$$\frac{\text{total-rotations}}{\text{total-time}} = \text{motor-speed}$$

Robot Algebra   Team: _____ Date: _____   17

ADJUSTING STRAIGHT SPEED – FINALIZING

Today, your team will finish adjusting straight speed on your Robot 1 and Robot 2 and will document your progress in a YouTube video.

Setting Up Your Robots

Make two dance programs, one program for each robot, that include your entire dance routine. Save them with a filename that includes your team letter, the letter of the robot, and "Sync7". For example, "Team**A**-R**D**-Sync**7**.c". Make sure to adjust the motor rotations and motor speed for the second robot so it goes the same distance in the same amount of time as your first robot. Feel free to use whatever solution you want to do this.

Your YouTube Progress Video

Every team will run their robots side-by-side with their music after adjusting the straight distances and speed. We will interview you in the video to find out what you have done. Here are the questions we will ask:

Before Running the Robots

1. Last time we saw your robots dancing you had adjusted the number of motor rotations on your second robot so that it went the same straight distance as your first robot, but the two robots were still way out of sync when you ran them side-by-side. What have you changed in the program since last time? Why?

2. What should I expect to see different from last time in terms of the robots' dancing being in sync?

After Running the Robots

3. Did the robots do what you thought they would do?

4. Are they in sync? If not, what do you think the problem is?

5. What do you need to do now to get them more in sync? How do you plan to do that?

APPENDIX E

MODEL ELICITING ENVIRONMENT ACTIVITY MATERIALS

Wksht 1 – Problem Introduction
Wksht 2 – Quality Assurance Guide
Wksht 3 – Programming Reference
Wksht 4 – Keeping Track
Wksht 5 – Team Ideas – Your First Ideas
Wksht 6 – Team Ideas – Synchronizing Straight Distance
Wksht 7 – Synchronizing Distance – Example Strategy 1
Wksht 8 – Synchronizing Distance – Example Strategy 1
Wksht 9 – Synchronizing Distance – Choosing the Best Strategy
Wksht 10 – Extension Problem
Wksht 11 – Team Ideas – Synchronizing Straight Timing
Wksht 12 – Synchronizing Timing – Example Strategies
Wksht 13 – Synchronizing Timing – Choosing the Best Strategy
Wksht 14 – Team Ideas – Synchronizing Turns
Wksht 15 – Synchronizing Turns – Example Strategy

Robot Synchronized Dancing

Bots-N-Sync is a robot dance team that specializes in doing synchronized dances—many robots doing the same dance moves at the same time. They are hugely popular thanks to the power of the Internet. They record videos of their routines and post them on YouTube. Although they have only completed two routines so far, both videos have gone viral with millions of views. The captain of the *Bots-N-Sync* team wants to make sure the team continues their success, but she needs your help.

Figure 1: Some current *Bots-N-Sync* dancers. All are Lego NXT robots.

The Problem

The team is growing a large and devoted fan base by encouraging their fans to submit dance routines online on the team's website. The problem is that each dance routine is designed for the team's original robot, *Beyonce*, but the robots on the dance team are all different. When the captain first downloads a dance routine to all the robots, each robot moves in different ways and they are definitely not in sync with each other. An example is shown on the next page with the latest submitted dance routine. In the past, when the team worked on just one dance routine at a time and with only their original team of robots, guess-and-check to adjust each move individually for each robot was tiresome but did work. Now, though, with routines being submitted each day and the increasing pressure from fans to put out fresh videos, *Bots-N-Sync* captain needs a much better solution.

Your Job

Create a "how to" toolkit that the *Bots-N-Sync* captain can use to modify submitted dance routine programs so that all of the dancers do the routines in sync with each other. New dance routines are submitted often and new dancers will be joining the team regularly. So, a good toolkit would work for the current dance routine, but an ideal toolkit would be easy to use or adapt for new routines and new robots. An ideal toolkit would also include explanations of why the solution works, so the captain can easily understand how it works and how it can be adapted later for other similar situations. Your toolkit can utilize words, numbers, graphs, pictures, and/or any other form that effectively conveys your ideas and meets the needs of your client, the *Bots-N-Sync* captain.

Robot Synchronized Dancing – Introduction

1

Current Dance Routine

The *Bots-N-Sync* captain is currently trying to modify the following dance routine, submitted by the fan *I_<3_BNS*. The captain wants you to use this routine as an example. The routine is set to the *Cupid Shuffle*. It has 10 moves that are listed in the table below.

|	Dance Routine				Beyonce Base Program Parameters	
	Movement Type	Target Distance	Target Turn	Target Time	Motor Rotations	Motor Speed
1	Straight Forward	47.1 cm		4.17 sec	5.00 rot	1.20 rot/sec
2	Straight Backward	75.4 cm		5.00 sec	8.00 rot	1.60 rot/sec
3	Point Turn Right		196 °	3.33 sec	3.00 rot	0.90 rot/sec
4	Point Turn Left		131 °	3.33 sec	2.00 rot	0.60 rot/sec
5	Straight Backward	22.6 cm		3.75 sec	2.40 rot	0.64 rot/sec
6	Straight Forward	30.2 cm		3.76 sec	3.20 rot	0.85 rot/sec
7	Swing Turn Left Forward		66 °	3.33 sec	2.00 rot	0.60 rot/sec
8	Swing Turn Right Forward		98 °	3.33 sec	3.00 rot	0.90 rot/sec
9	Swing Turn Left Backward		131 °	3.33 sec	4.00 rot	1.20 rot/sec
10	Swing Turn Right Backward		164 °	3.33 sec	5.00 rot	1.50 rot/sec
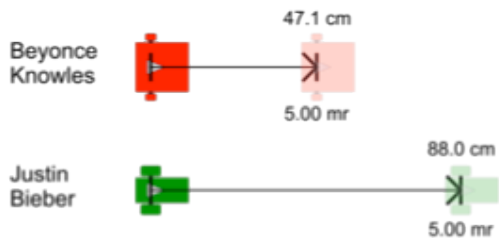
Figure 2: This is what happens when the *Bots-N-Sync* captain puts the routine directly on a second robot without making adjustments. The robots are way out of sync even after the first move!

"How To" Toolkit Quality Assurance Guide

2

Self Assessment

The tables below can be used to assess your solution for the *Robot Synchronized Dancing* toolkit. Good solutions meet the immediate needs of the client. The best solutions are also easy to understand and adapt to similar situations.

Create a "how to" toolkit that the Bots-N-Sync captain can use to modify submitted dance routine programs so that all of the dancers do the routines in sync with each other.

Needs of the Client

Performance Level	Sharable or Reusable	Useful for this Specific Situation	Requires only Minor Editing	Requires Major Extensions or Refinements	Requires Redirection
NEEDS OF THE CLIENT Does the solution meet the needs of the client?	This tool not only works for the immediate situation, but it also would be easy for others to modify and use it in similar situations.	No changes will be needed to meet the immediate needs of the client.	This product is nearly ready to be used. It still needs a few small modifications, additions, or refinements.	The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.	The product is on the wrong track. Indicates a lack of understanding about the problem or a lack of useful ideas about how to solve it.
What might the client say?	"Excellent, this tool will be easy for me to modify or use in other similar situations – when the data are slightly different."	"Ahhh, this will work well as it is. I won't even need to do any editing."	"Hmmm, this is close to what I need. With a few small changes or additions, it would do the job."	"You're on the right track, but this still needs a lot more work before it'll be in a form that's useful."	"Start over. This won't work. Think about it differently. Use different ideas or procedures."

WKSHT 2 – QUALITY ASSURANCE GUIDE
(PAGE 2 OF 2)

2

Self Assessment (cont.)

The tables below can be used to assess your solution for the *Robot Synchronized Dancing* toolkit. Good solutions meet the immediate needs of the client. The best solutions are also easy to understand and adapt to similar situations.

Create a "how to" toolkit that the Bots-N-Sync captain can use to modify submitted dance routine programs so that all of the dancers do the routines in sync with each other.

Letter to the Client

Performance Level	Sharable or Reusable	Useful for this Specific Situation	Requires only Minor Editing	Requires Major Extensions or Refinements	Requires Redirection
LETTER TO THE CLIENT Does the letter completely explain the solution, about why it works, how/why it works, and how it could be adapted?	The letter provides enough detail for the client to implement the suggested solution, and it includes information about why it works, and how to alter the solution for different but similar circumstances.	The letter provides enough detail for the client to implement the suggested solution without additions or clarification.	The letter provides enough detail that the client could implement the procedure with only minor clarification.	The letter only describes the solution process generally. The client would be unable to implement the solution process simply from the information provided in the letter. The client would need clarification, more information, or help.	The letter describes very little of the solution process.

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3

Dance Programming Quick Reference

The Basics

To do the dance programming you will use ROBOTC, a powerful C-based programming environment designed specifically for robot programming. Similar to any other programming environment, ROBOTC controls the robot by executing a sequence of commands from a program. In the dancing programs, the majority of commands are for dance movements, such as moving straight forward or making a swing turn. Every dance movement command takes two parameters. The first one indicates how many times to turn the motors, or *motor rotations* (in rotations), and the second indicates how fast to turn the motors, or *motor speed* (in rotations per second). The motor speed is limited to 2.0 rotations per second or less. For example:

```
straightBackward(3.0, 1.0); // This robot will move backward for
                            // 3 rotations at 1 rotation per second.
```

Your can modify those dance movement parameters for each dance move to tell the robot to do more or less motor rotations or to rotate the motors faster or slower.

A Programming Tip – Simplifying the Problem

This problem is difficult and has many parts. It would be wise to focus on simplifying the problem, and trying to solve that aspect well before moving on to other parts. One way to do this is to focus on only one move at a time or on only certain types of moves. We have provided you with a function that makes this easier to do:

```
setMoveMode(MMOneMoveAtATime, MMStraightsOnly);
```

Starter Programs

To help you get started, a number of programs have been downloaded to Beyonce.

Program Name	Description
B-Dance	The original dance routine. Completes *all the movements, continuously.*
B-1Mv-All	Completes *all the movements*, and does them *one-at-a-time.*
B-1Mv-Str	Completes *only the straight movements*, and does them *one-at-a-time.*
B-1Mv-Trn	Completes *only the turn movements*, and does them *one-at-a-time.*
B-Cnt-Str	Completes *only the straight movements*, and does them *continuously.*
B-Cnt-Trn	Completes *only the turn movements*, and does them *continuously.*

Robot Synchronized Dancing – Dance Programming Quick Reference

3

File Menu
To open a program or save a program as a new filename (e.g., "-v2")

Robot Menu
To compile and download programs to the robot

Set Move Mode
Uncomment to run one move at a time or only a certain move type (e.g., just straight moves)

Comment
"// ..."
Ignored by the program

Dance Move Sequence
Executed in order one after the other

```
18
19    task main ()
20    {
21        // ----- Modify this line with your robot nam
22        initializeRobot(RMJustinTimberlake)
23
24        //setMoveMode(MMOneMoveAtATime, MMAllMoveTyp
25        //setMoveMode(MMOneMoveAtATime, MMStraightsO
26
27        movesStart();
28
29        // ----- Edit the dance moves BELOW this line -----
30
31        straightForward (5.00, 1.20);
32        straightBackward (8.00, 1.60);
33        turnPointRight (3.00, 0.90);
34        turnPointLeft (2.00, 0.60);
35        straightBackward (2.40, 0.64);
36        straightForward (3.20, 0.85);
37        turnSwingLeftForward (2.00, 0.60);
38        turnSwingRightForward (3.00, 0.90);
39        turnSwingLeftBackward (4.00, 1.20);
40        turnSwingRightBackward (5.00, 1.50);
41
42        // ----- Edit the dance moves ABOVE this line -----
43
```

Parameter 1
Motor Rotations
(rotations)

Parameter 2
Motor Speed
(rotations per second)

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Robot Synchronized Dancing – Keeping Track

4

Keeping Track of Justin's Program Parameters

To help you keep track of the program values that work for Justin, you can write them in the following table. You can also use the space below and on the back to do calculations.

#	Movement Type	ALL ROBOTS Dance Routine Specification			BEYONCE Original Program Parameters		JUSTIN BIEBER Dance Moves when using the Original Program Parameters			JUSTIN BIEBER Adjusted Program Parameters	
		Target Distance	Target Turn	Target Time	Motor Rotations	Motor Speed	Distance	Turn	Time	Motor Rotations	Motor Speed
1	Straight Forward	47.1 cm		4.17 sec	5.00 mr	1.20 mr/sec	88.0 cm		4.17 sec		
2	Straight Backward	75.4 cm		5.00 sec	8.00 mr	1.60 mr/sec	140.7 cm		5.00 sec		
3	Point Turn Right		196 °	3.33 sec	3.00 mr	0.90 mr/sec		526 °	3.33 sec		
4	Point Turn Left		131 °	3.33 sec	2.00 mr	0.60 mr/sec		351 °	3.33 sec		
5	Straight Backward	22.6 cm		3.75 sec	2.40 mr	0.64 mr/sec	42.2 cm		3.75 sec		
6	Straight Forward	30.2 cm		3.76 sec	3.20 mr	0.85 mr/sec	56.3 cm		3.76 sec		
7	Swing Turn Left Forward		66 °	3.33 sec	2.00 mr	0.60 mr/sec		175 °	3.33 sec		
8	Swing Turn Right Forward		98 °	3.33 sec	3.00 mr	0.90 mr/sec		263 °	3.33 sec		
9	Swing Turn Left Backward		131 °	3.33 sec	4.00 mr	1.20 mr/sec		351 °	3.33 sec		
10	Swing Turn Right Backward		164 °	3.33 sec	5.00 mr	1.50 mr/sec		438 °	3.33 sec		

250

Robot Synchronized Dancing – Your First Ideas

5

Team Name: _____   Team Members: _____

Team Ideas – Your First Ideas

Use this space to write out your thoughts, to do estimates or calculations, to draw pictures that illustrate your ideas, or any other notes that will capture your ideas so you can remember them and share them with the rest of us. Do not erase any of your work. Instead, use other blank space below or on the back.

Create a "how to" toolkit that the Bots-N-Sync *captain can use to modify submitted dance routine programs so that all of the dancers do the routines in sync with each other.*

*Talk as a group and try to come up with some **initial ideas** about how to solve this problem for the dance team captain.*

RA – Robotics Academy & LRDC

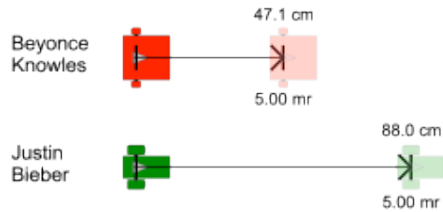Robot Synchronized Dancing – Sync Straight Distance

6

Team Name: _____   Team Members: _____

Team Ideas – Synchronizing Straight Distance

Use this space to write out your thoughts, to do estimates or calculations, to draw pictures that illustrate your ideas, or any other notes that will capture your ideas so you can remember them and share them with the rest of us. Do not erase any of your work. Instead, use other blank space below or on the back.

Create a "how to" toolkit that the Bots-N-Sync *captain can use to modify submitted dance routine programs so that all of the dancers do the routines in sync with each other.*

*Focus here on how to get robots to have **synchronized straight distances** – two robots going the same distance forward or backward.*
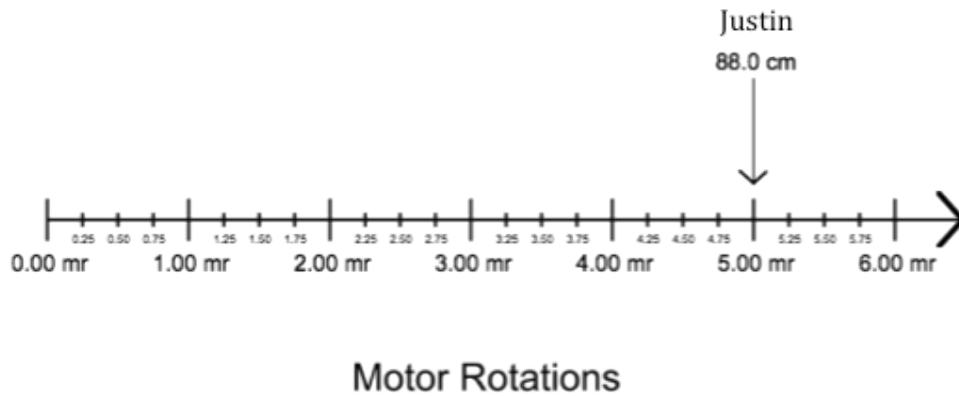
Beyonce Knowles — 47.1 cm — 5.00 mr

Justin Bieber — 88.0 cm — 5.00 mr

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Robot Synchronized Dancing – Sync Straight Distance

6

Team Name: _____   Team Members: _____

Dance Move #	Correct Distance	Beyonce's Working Motor Rotations	Justin's Incorrect Distance (Distance Justin moves using Beyonce's MRs)	Justin's Working Motor Rotations
1	47.1cm	5.00mr	88.0cm	
2	75.4cm	8.00mr	140.8cm	
5	22.6cm	2.40mr	42.2cm	
6	30.2cm	3.20mr	56.3cm	
	D	MR_B	$D_{Incorrect}$	MR_J

Justin

88.0 cm

0.00 mr	1.00 mr	2.00 mr	3.00 mr	4.00 mr	5.00 mr	6.00 mr

0.25 0.50 0.75   1.25 1.50 1.75   2.25 2.50 2.75   3.25 3.50 3.75   4.25 4.50 4.75   5.25 5.50 5.75

Motor Rotations

Robot Synchronized Dancing – Example Strategy

7

Team Name: _____

Synchronizing Distance – Example Strategy

LRDC 06/14/10   Team Black – Eli

Eli's "Keep Adding/Subtracting 0.1" Strategy

My Solution for Getting My Robot to Go the Correct Distance

If my robot doesn't move far enough, then I add a little bit, 0.1mr, to the motor rotations and try again. If it still doesn't move far enough, then add a little bit again and try it on the robot again. I keep on adding a little bit and trying it out until the robot goes the right distance.

Why the Solution Works

When I increase the value for the more motor rotations, the distance that the robot moves straight also increases. So eventually, I will get the correct distance.

For Example

Any Move → MR_J = [keep adding on 0.1s]

Try #	Motor Rotations	Distance
1	1.00mr	9.4cm
2	1.10mr	10.4cm
3	1.20mr	11.3cm
4	1.30mr	12.3cm
. . .	. . .	
149	15.90mr	150.0cm

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Robot Synchronized Dancing – Example Strategy

8

Team Name: _____

Synchronizing Distance – Example Strategy – Afternoon Group

LRDC 06/15/10   Team Black – Eli

Eli's "Distance Scale Factor" Strategy

My Solution

Scale down the motor rotations that work for Beyonce to get the motor rotations that work for Justin by multiplying by a **scale factor**. This will work on every straight move in the dance routine.

$$\text{Any Move} \rightarrow MR_J = MR_B \times \text{Scale Factor}$$

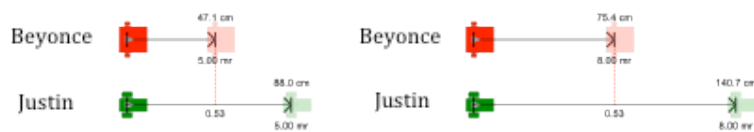$$\text{Move 1} \rightarrow MR_J(47.1cm) = 5.00mr \times 0.53 = 2.65mr$$

straightForward (5.00, 1.20)   straightForward (2.65, 1.20)

$$\text{Move 2} \rightarrow MR_J(75.4cm) = 8.00mr \times 0.53 = 4.24mr$$

straightBackward (8.00, 1.60)   straightBackward (4.24, 1.60)

Why It Works

Every time that we program the same motor rotations on both robots, Justin goes further than Beyonce by the same **relative amount**.

$$\text{Move 1} \rightarrow DIST_B \div DIST_J = 47.1cm \div 88.0cm = 0.53$$

$$\text{Move 2} \rightarrow DIST_B \div DIST_J = 75.4cm \div 140.7cm = 0.53$$

It's always the same scale!   Scale Factor = 0.53

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Robot Synchronized Dancing – Example Strategies

8

Team Name: _____

Synchronizing Distance – Example Strategy– Afternoon Group

LRDC 06/15/10   Team Black – Eli

Eli's "Half + Adjustment" Strategy

My Solution

Scale down the motor rotations that work for Justin to get the motor rotations that work for Beyonce by dividing her motor rotations in half and then add an adjustment of 0.15 motor rotations.

$$\text{Any Move} \rightarrow MR_J = MR_B + 2 + 0.15mr$$

$$\text{Move 1} \rightarrow MR_J(47.1cm) = 5.00mr + 2 + 0.15mr$$

$$MR_J(47.1cm) = 2.65mr$$

straightForward (5.00, 1.20)   straightForward (2.65, 1.20)

Why It Works

The distance Beyonce moves straight is always about half the distance Justin moves when they have the same amount of motor rotations. So Justin should only do half as many motor rotations as Beyonce. But since that isn't quite right, we have to add a little bit to Justin's motor rotations to get the correct distance.

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Robot Synchronized Dancing – Choosing the Best Strategy

9

Student Name: _____   Date: _____

Synchronizing Distance – Choosing the Best Strategy for You

Consider all the strategies that we have discussed so far. Choose the strategy that you think is the **best** one for getting different robots to move the same straight distance. In the space below, write down the name of the strategy below and state at least two reasons for choosing that one.

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Robot Synchronized Dancing – Choosing the Best Strategy

9

Student Name: _____   Date: _____

Synchronizing Distance – Using Your Best Strategy

Now use the strategy you chose as the best for you. Suppose Beyonce moved forward 300 centimeters when programmed to do 31.91 motor rotations. Trying that same motor rotations on Justin Bieber, he moves forward 562 centimeters. How many motor rotations would you try first for Justin Bieber to get him to move forward the same distance as Beyonce. Write down the number of motor rotations for Justin and describe how you got that value.

Robot: __Beyonce__   Robot: __Justin Bieber__

Distance: __300 cm__   Distance: __300 cm__

Motor Rotations: __31.91 mr__   Motor Rotations: _____

How did you get that value?

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Robot Synchronized Dancing – Sync Straight Distance

10

Team Name: _____   Team Members: _____

What if we have a new robot we haven't seen before?

Talk about this problem with your team, consider using or adapting any of the synchronizing distance strategies that we have discussed in class so far, and generate a solution.

> *Imagine we were adding a new robot to the dance team, but we haven't seen the robot yet. We do know that when the robot was programmed to do 7.50 motor rotations it moved forward 192 cm. How far forward would the robot move if it were reprogrammed to do 6.25 motor rotations?*

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Robot Synchronized Dancing – Sync Straight Timing

<div style="border:1px solid">11</div>

Team Name: _____   Team Members: _____

Team Ideas – Synchronizing Straight Timing

In the space below, keep track of your ideas as you develop a solution for the *Robot Synchronized Dancing* problem. Use this space to write out your thoughts, to do estimates or calculations, to draw out ideas, or any other notes that will capture your thinking as your work through the problem. Do not erase any of your work. Instead, use other blank space and indicate how your newer ideas improve on your older ones.

Create a "how to" toolkit that the Bots-N-Sync *captain can use to modify submitted dance routine programs so that all of the dancers do the routines in sync with each other.*

47.1 cm in **4.17 sec**

Beyonce

5.00 mr at **1.20 mrps**

Focus here on how to get robots to have **synchronized straight timing** *– two robots going the same distance in the same amount of time.*
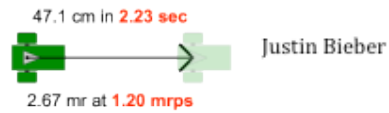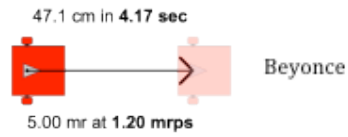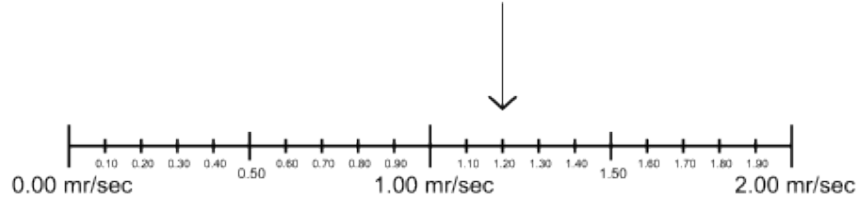
47.1 cm in **2.23 sec**

Justin Bieber

2.67 mr at **1.20 mrps**

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Robot Synchronized Dancing – Sync Straight Timing

Team Name: _____   Team Members: _____

Justin Bieber
2.67 mr in 2.23 sec

0.00 mr/sec   1.00 mr/sec   2.00 mr/sec

Motor Rotations per Second

Dance Move #	Correct Distance	Correct Time	Beyonce's Working Values	Justin's Working Motor Rotations	Justin's Working Motor Speed
1	47.1cm	4.17sec	5.00mr, 1.20mr/sec	2.67mr	
2	75.4cm	5.00sec	8.00mr, 1.60mr/sec	4.28mr	
5	22.6cm	3.75sec	2.40mr, 0.64mr/sec	1.28mr	
6	30.2cm	3.76sec	3.20mr, 0.85mr/sec	1.71mr	
	D	T	MR_{JT}, MS_{JT}	MR_{BE}	MS_{BE}

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Robot Synchronized Dancing – Example Strategy

12

Team Name: _____

Synchronizing Timing – Example Strategy – Afternoon Group

LRDC 06/16/10   Team Black – Eli

Eli's "Rotations-in-1-Second" Strategy

My Solution for Synchronizing Timing

First, figure out the correct number of motor rotations for the robot to go the correct distance. Then, divide the correct number of motor rotations by the target time for that move.
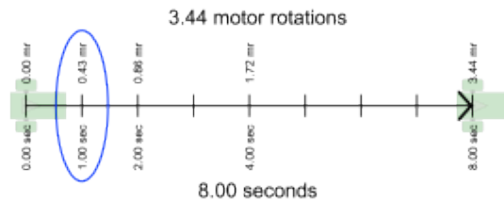
```
Any Move → MtrSpeed = MtrRotations ÷ TargetTime
```

$$\text{Move 1 (47.1cm, 4.17sec)} \rightarrow$$
$$\text{MtrSpeed}_{Justin} = 2.67mr \div 4.17sec = 0.64mr/sec$$

```
straightForward (2.67, 0.64)
```

Why It Works

The motor speed is the same as a unit rate for the amount of motor rotations needed in 1 second. By dividing the total number of motor rotations by the total time, I can get the *Rotations-in-1-Second*, which I can use as that move's motor speed.

```
Example - Move Justin 60cm (3.44mr) in 8.00sec
```

$$3.44mr \div 8.00sec = 0.43mr/sec$$

3.44 motor rotations

8.00 seconds

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Robot Synchronized Dancing – Example Strategies

Team Name: _____

| 12 |

Synchronizing Timing – Example Strategy – Afternoon Group

LRDC 06/16/10   Team Black – Eli

Eli's "D=RT Equation" Strategy

My Solution for Synchronizing Timing

Use the standard distance equals rate times time equation. Take the target distance divide that by the target time and that should give you the correct speed.

$$\text{Any Move} \rightarrow \text{Speed} = \text{TargetDistance} \div \text{TargetTime}$$

$$\text{Move 1 (47.1cm, 4.17sec)} \rightarrow$$

$$\text{Speed}_{\text{Justin}} = 47.1\text{cm} \div 4.17\text{sec} = 11.29\text{cm/sec}$$

```
straightForward (2.67, 11.29)
```

Why It Works

The robot is like any other object that moves at a constant speed, so it should follow the same rules.

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Robot Synchronized Dancing – Choosing the Best Strategy

13

Student Name: _____   Date: _____

Synchronizing Timing – Choosing the Best Strategy for You

Consider all the strategies that we have discussed so far. Choose the strategy that you think is the **best** one for getting different robots to move the same straight distance in the same amount of time. In the space below, write down the name of the strategy below and state at least two reasons for choosing that one.

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Robot Synchronized Dancing – Choosing the Best Strategy

| 13 |

Student Name: _____   Date: _____

Synchronizing Timing – Using Your Best Strategy

Now use the strategy you chose as the best for you. You can adapt it to fit this situation.

A first robot moved forward a distance of 120 cm in 4.0 seconds. Its program had it do 6.00 motor rotations at 1.50 motor rotations per second. A programmer needs to make a second robot do the same distance in the same amount of time. First, she got the second robot to go the same distance by changing the motor rotations to 2.00 motor rotations. But then the robot completed the move in 3.0 seconds.

1. Draw a picture of the situation.

2. What should she change the motor speed to so that the second robot does the move in the correct amount of time? (Show your work)

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Robot Synchronized Dancing – Sync Turns

Team Name: _____   Team Members: _____

Team Ideas – Synchronizing Turns

Use this space to write out your thoughts, to do estimates or calculations, to draw pictures that illustrate your ideas, or any other notes that will capture your ideas so you can remember them and share them with the rest of us. Do not erase any of your work. Instead, use other blank space below or on the back.

Create a "how to" toolkit that the Bots-N-Sync captain can use to modify submitted dance routine programs so that all of the dancers do the routines in sync with each other.

*Focus here on how to get robots to have **synchronized turns**. First, get them to turn the same amount, and then also in the same time.*

196 deg   526 deg

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Robot Synchronized Dancing – Sync Turns

14

Team Name: _____   Team Members: _____

Dance Move #	Correct Turn	Correct Time	Beyonce's Working Values	Justin's Working Motor Rotations	Justin's Working Motor Speed
3	196°	3.33sec	3.00mr, 0.90mr/sec		
4	131°	3.33sec	2.00mr, 0.60mr/sec		
7	66°	3.33sec	2.00mr, 0.60mr/sec		
8	98°	3.33sec	3.00mr, 0.90mr/sec		
9	131°	3.33sec	4.00mr, 1.20mr/sec		
10	164°	3.33sec	5.00mr, 1.50mr/sec		
	A	T	$MR_{Beyonce}$, $MS_{Beyonce}$	MR_{Justin}	MS_{Justin}

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Robot Synchronized Dancing – Example Strategy

15

Team Name: _____

Synchronizing Turns – Example Strategy – Afternoon Group

LRDC 06/17/10   Team Black – Eli

Eli's "Turns Scale Factor" Strategy

My Solution for Synchronizing Turning Angles
First, figure out the scale factor that relates how far the new robot turns compared to the original robot when using the same amount of motor rotations.

Turns Scale Factor →

$$SF_{Turns} = Deg_B \div Deg_J = 196deg \div 526deg = 0.37$$

$$Any\ Move \rightarrow MR_{Justin} = MR_{Beyonce} \times SF_{Turns}$$

$$Move\ 3 \rightarrow MR_{Justin}(196deg) = 3.00mr \times 0.37 = 1.11mr$$

turnPointRight (1.11, 0.90)

Why It Works
Some robots take longer to turn, but it is always the same relative amount compared to a different robot. So once you figure out how much more (or less) one robot will turn than another per motor rotation, you can figure out how to scale the motor rotations for the original robot to the correct motor rotations for the new robot.

1mr = 9.4cm

1mr = 17.6cm

16.5cm

11.5cm

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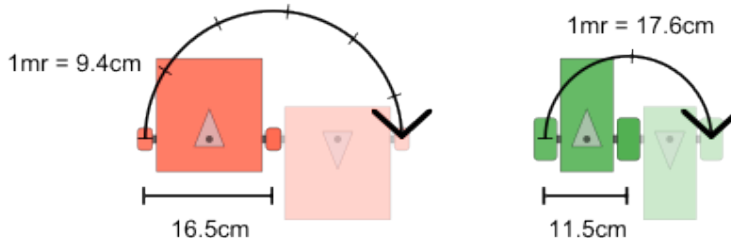
APPENDIX F

MECHANISTIC ENVIRONMENT ACTIVITY MATERIALS

Wksht 7 – Synchronizing Distance – Example Strategy 1
Wksht 8 – Synchronizing Distance – Example Strategy 1
Wksht 12 – Synchronizing Timing – Example Strategies
Wksht 15 – Synchronizing Turns – Example Strategy
Wksht 16 – Madness Competition

Robot Synchronized Dancing – Example Strategy

7

Team Name: _____

Synchronizing Distance – Example Strategy

LRDC 06/14/10   Team Black – Eli

Eli's "Keep Adding/Subtracting 0.1" Strategy

My Solution for Getting My Robot to Go the Correct Distance

If my robot doesn't move far enough, then I add a little bit, 0.1mr, to the motor rotations and try again. If it still doesn't move far enough, then add a little bit again and try it on the robot again. I keep on adding a little bit and trying it out until the robot goes the right distance.

Why the Solution Works

The motors turn the wheels and when the wheels turn, the robot moves. By making the wheels turn more, the distance that the robot moves also increases. So eventually, by increasing the wheel turns enough, I will get the correct distance.

For Example

Any Move → MR_J = [keep adding on 0.1s]

Try #	Motor Rotations	Distance
1	1.00mr	9.4cm
2	1.10mr	10.4cm
3	1.20mr	11.3cm
4	1.30mr	12.3cm
...	...	
149	15.90mr	150.0cm

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Robot Synchronized Dancing – Example Strategy

8

Team Name: _____

Synchronizing Distance – Example Strategy – Morning Group

LRDC 06/15/10   Team Black – Eli

Eli's "Wheel Size Scale Factor" Strategy

My Solution

Scale down the motor rotations that work for Beyonce to get the motor rotations that work for Justin by multiplying by a *scale factor*. This will work on every straight move in the dance routine.

$$\text{Any Move} \rightarrow MR_J = MR_B \times \text{Scale Factor}$$

$$\text{Move 1} \rightarrow MR_J(47.1cm) = 5.00mr \times 0.53 = 2.65mr$$

straightForward (5.00, 1.20)   straightForward (2.65, 1.20)

$$\text{Move 2} \rightarrow MR_J(75.4cm) = 8.00mr \times 0.53 = 4.24mr$$

straightBackward (8.00, 1.60)   straightBackward (4.24, 1.60)

Why It Works

The circumference of Beyonce's wheels are smaller than Justin's, so Justin should only do a part of a motor rotation every time Beyonce does a complete motor rotation.

$$\text{Move 1} \rightarrow \text{Wheel}_B \div \text{Wheel}_J = 9.42cm \div 17.58cm = 0.53$$

It's the ratio of the *wheel circumferences*!   Scale Factor = 0.53

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Robot Synchronized Dancing – Example Strategies

8

Team Name: _____

Synchronizing Distance – Example Strategy– Morning Group

LRDC 06/15/10   Team Black – Eli

Eli's "Half + Adjustment" Strategy

My Solution

Scale down the motor rotations that work for Justin to get the motor rotations that work for Beyonce by dividing her motor rotations in half.

$$\text{Any Move} \rightarrow \text{MR}_J = \text{MR}_B + 2 + 0.15\text{mr}$$

$$\text{Move 1} \rightarrow \text{MR}_J(47.1\text{cm}) = 5.00\text{mr} + 2 + 0.15\text{mr}$$

$$\text{MR}_J(47.1\text{cm}) = 2.65\text{mr}$$

straightForward (5.00, 1.20)   straightForward (2.65, 1.20)

Why It Works

The circumference of Beyonce's wheels are about half of Justin's, so Justin should only do half as many motor rotations as Beyonce. But since that isn't quite right, we have to add a little bit to Justin's motor rotations to get the correct distance.

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Robot Synchronized Dancing – Example Strategy

12

Team Name: _____

Synchronizing Timing – Example Strategy – Morning Group

LRDC 06/16/10   Team Black – Eli

Eli's **"Rotations-in-1-Second"** Strategy

My Solution for Synchronizing Timing

First, figure out the correct number of motor rotations for the robot to go the correct distance. Then, divide the number of motor rotations by the target time for that move.

Any Move → MtrSpeed = MtrRotations ÷ TargetTime

Move 1 (47.1cm, 4.17sec) →
MtrSpeed$_{Justin}$= 2.67mr ÷ 4.17sec = 0.64mr/sec

straightForward (2.67, 0.64)

Why It Works

The size of the wheel is key for determining the number of rotations for a specific distance, but for timing, wheel size matters only indirectly. **The number of rotations is what really matters**. The motor speed is the same as figuring out how many *motor rotations* we need to complete in *1 second*. Dividing the total motor rotations by the total time is like sectioning the total time into 1-second chunks.

Example – Move Justin 60cm (3.44mr) in 8.00sec
3.44mr ÷ 8.00sec = 0.43mr/sec
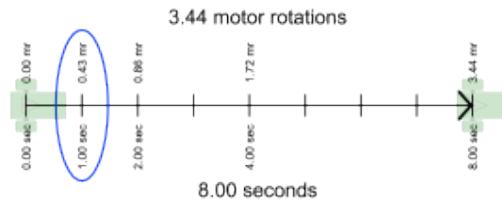
3.44 motor rotations

8.00 seconds

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Robot Synchronized Dancing – Example Strategies

12

Team Name: _____

Synchronizing Timing – Example Strategy – Morning Group

LRDC 06/16/10   Team Black – Eli

Eli's **"D=RT Equation"** Strategy

My Solution for Synchronizing Timing

Use the standard distance equals rate times time equation. Take the target distance divide that by the target time and that should give you the correct speed.

Any Move → Speed = TargetDistance ÷ TargetTime

Move 1 (47.1cm, 4.17sec) →
Speed$_{Justin}$= 47.1cm ÷ 4.17sec = 11.29cm/sec

straightForward (2.67, 11.29)

Why It Works

The robot is like any other object that moves at a constant speed, so it should follow the same rules.

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Robot Synchronized Dancing – Example Strategy

15

Team Name: _____

Synchronizing Turns – Example Strategy – Morning Group

LRDC 06/17/10   Team Black – Eli

Eli's "Robot Width Scale Factor" Strategy

My Solution for Synchronizing Turning Angles
First, figure out the scale factor between the wheel sizes (the distance around the wheel). Then, figure out the scale factor between the robot widths (the distance between the wheels). Then multiply those two scale factors to get one scale factor that will adjust the motor rotations correctly for any turn.

$$\text{Any Move} \rightarrow MR_{Justin} = MR_{Beyonce} \times SF_{Turns}$$

$$\text{Wheels} \rightarrow SF_{WS} = WS_{Beyonce} \div WS_{Justin} = 9.4cm \div 17.6cm = 0.53$$
$$\text{Width} \rightarrow SF_{RW} = RW_{Justin} \div RW_{Beyonce} = 11.5cm \div 16.5cm = 0.70$$

$$\text{Scale Factor} \rightarrow SF_{Turns} = SF_{WS} \times SF_{RW} = 0.37$$

$$\text{Move 3} \rightarrow MR_{Justin}(196deg) = 3.00mr \times 0.37 = 1.11mr$$

turnPointRight (1.11, 0.90)

Why It Works
The amount the robot turns depends on two things – (1) *the size of the robot's wheels* and (2) *the distance between the robot's wheels*. When the width of the robot is smaller, it doesn't have to go as far to turn the same amount, which means it will need less motor rotations.
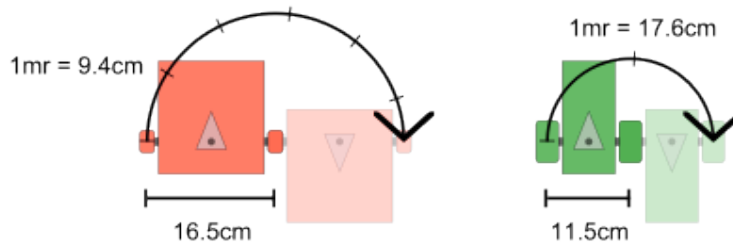
1mr = 9.4cm   1mr = 17.6cm

16.5cm   11.5cm

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16

Team Name: _____   Team Members: _____

Madness Game

Scoring

1. *End Zone Balls* - A team gets 3 points for every ping pong ball that is in the *End Zone Scoring Area* at the end of the round or that was removed from the *End Zone Scoring Area* during the round.
2. *Loose Balls* - A team gets 1 point for every ping pong ball that is freed from its toilet paper tube, but that is not in the *End Zone Scoring Area* at the end of the round.
3. *Robot Return Penalties* – A team gets a 1 point penalty for touching its robot if no part of the robot is in the *Starting Position Area*.
4. *Total Score* - Scoring is tabulated at the end of the round, so ping pong balls that at one point made it to the *End Zone Scoring Area*, but did not stay there or were not removed by a team member, will not be counted.
5. *Final Score* - There will be three rounds for each team. The max score from the three rounds for each team will count as that team's final score for the competition.

Rules

1. A round lasts 60 seconds.
2. "Starting Position" is with the entire robot completely within the base. The base is defined as the center section on the one end of the board.
3. The robot is to leave the base, perform challenges and return to base with only its program to control it. If the team touches the robot before it returns to base a "Robot Return Penalty" will be given and the robot must be returned to "Starting Position."
4. A successful robot return is noted when any part of the robot crosses the base line.
5. If the robot is retrieving an object back to the *End Zone Scoring Area*, a team member may grab and remove the object as soon as the object is entirely within the area, no matter where the robot is.
6. While the robot is in "Starting Position", the team may repair it, reposition it, load or unload it, or change the program.
7. Robots will be programmed using ROBOTC and the provided library of functions.
8. If a team member touches the robot as it is returning an object and before the object is entirely within the end zone area, the object is returned immediately to its original position.
9. Teams may bring up to two robots (only one on the gameboard at a time).
10. Playing field objects will remain as the robot leaves them and will not be reset during the game.
11. Challenges may be tried in parts, in any order, more than one at a time or skipped.
12. Only the team members of the team currently running may be at the table at one time. Everyone else must stay back so as not to interrupt the game.
13. The referee makes all field judgment calls and can not consider protests.

Robot Synchronized Dancing – Madness Game

16

Team Name: _____   Team Members: _____

Working with the Robots

Everyone will be using the same robots for this game. We only have two robots, so you will only have limited time with the robots. It will be important that you plan your strategy out and continue working on your solution even when you don't have access to a robot.

The Red Team and the Purple Team will work with the same robot. The Purple team will get it first, then the Red team. They will switch back and forth every 5 minutes during the planning period.

The Blue Team and the Green Team will work with the same robot. The Blue team will get it first, then the Green team. They will switch back and forth every 5 minutes during the planning period.

RSD – Robotics Academy & LRDC

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