

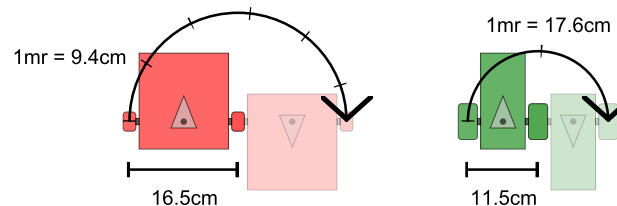
Resources for Learning Robots: Environments and Framings Connecting Math in Robotics



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Cognitive Studies in Education
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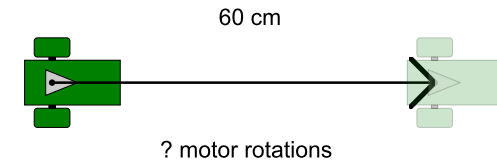


$$\begin{aligned}
 &X = \text{motor rotations} \\
 &Y = \text{desired degree} \\
 &\frac{1 \text{ motor rotation}}{175.3^\circ \text{ per rotation}} = \frac{X}{Y} \\
 &\Leftrightarrow X \\
 &\frac{1 \text{ mr}}{175.3^\circ} = \frac{X}{140^\circ} \\
 &140^\circ = \frac{140^\circ}{175.3} = \frac{175.3^\circ}{175.3} \cdot 2 \\
 &1.12 = \frac{2}{X}
 \end{aligned}$$



Why robots?

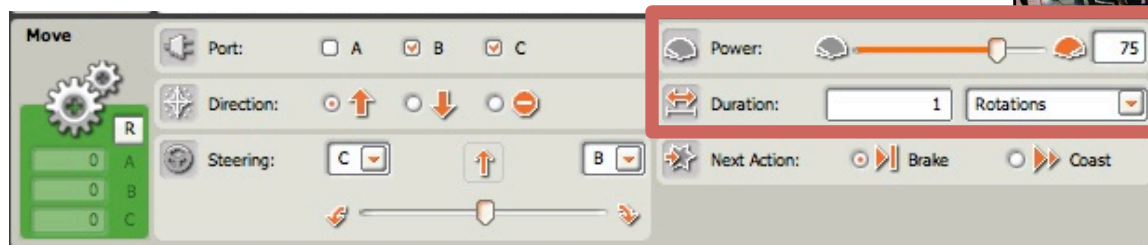
Controlling robot movements



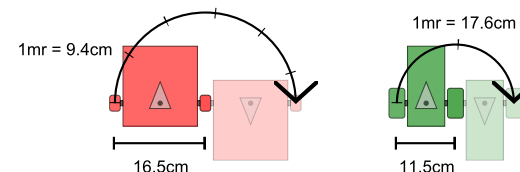
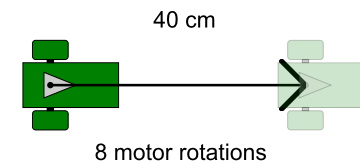
- Popular & engaging context for integrated STEM problem solving
- Knowledge-rich context (Schauble, 1996)
 - Inspectable and manipulable (Rozenblit & Keil, 2002)



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- Quantifiable (reliable patterns) $\text{Distance} = \text{Motor Rotations} \times \text{Wheel Circumference}$
- Non-math strategies common (Lannin, 2005)
- Connects to proportional reasoning (Lamon, 2007)
 - Well-studied in-school (Ben-chaim et al., 1998) and out-of-school (Hoyles et al., 2001; Schliemann et al., 1992)
 - Opportunities for extending the math



Part 1 – Introduction

Problems and Resources

(Engle & Conant, 2002)



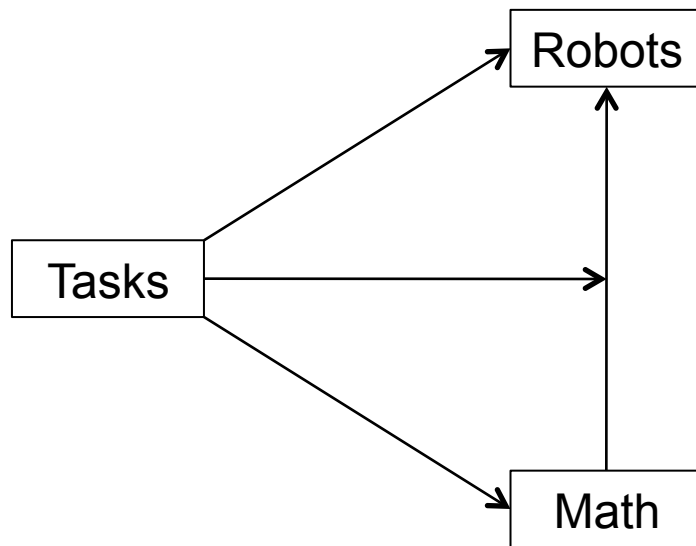
What features of **learning environment tasks** impact the ways in which participants **engage with** and **learn to use** the mathematical structure of physical situations?

Develop
interest in and value of
math in the situation

Develop
sophisticated ideas for using
math in the situation

Part 1 – Introduction

What **features of learning environment tasks** impact the ways in which participants engage with and learn to use the mathematical structure of physical situations?



Alternative **alignments** (Krajcik et al., 2008)
of tasks to math-to-robot content
(observational and design methods)



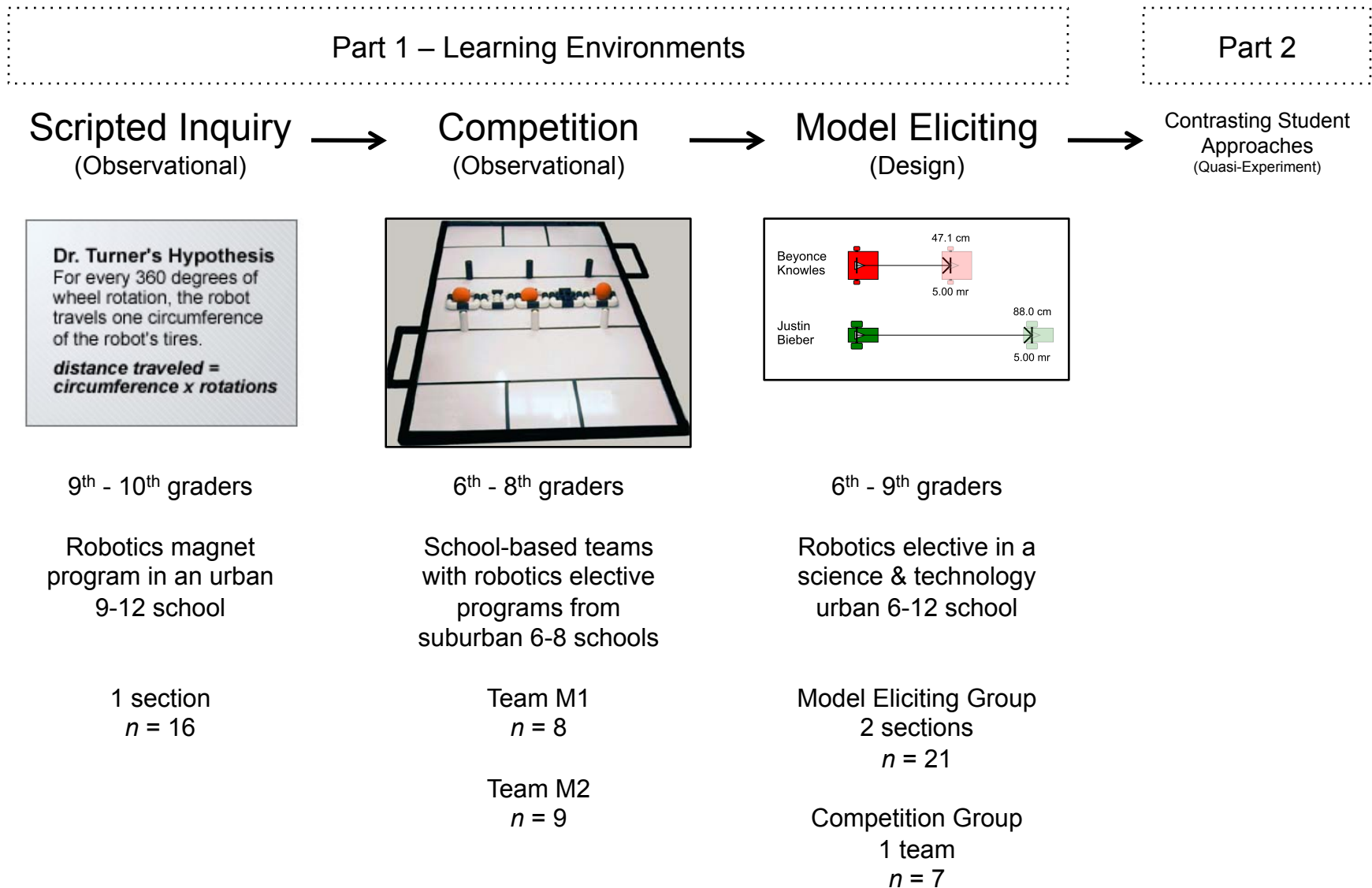
Claim that less productive alignments are:

- a) align to a **math idea** using robots as the context
- b) align to a **robot problem** that has opportunities to use math

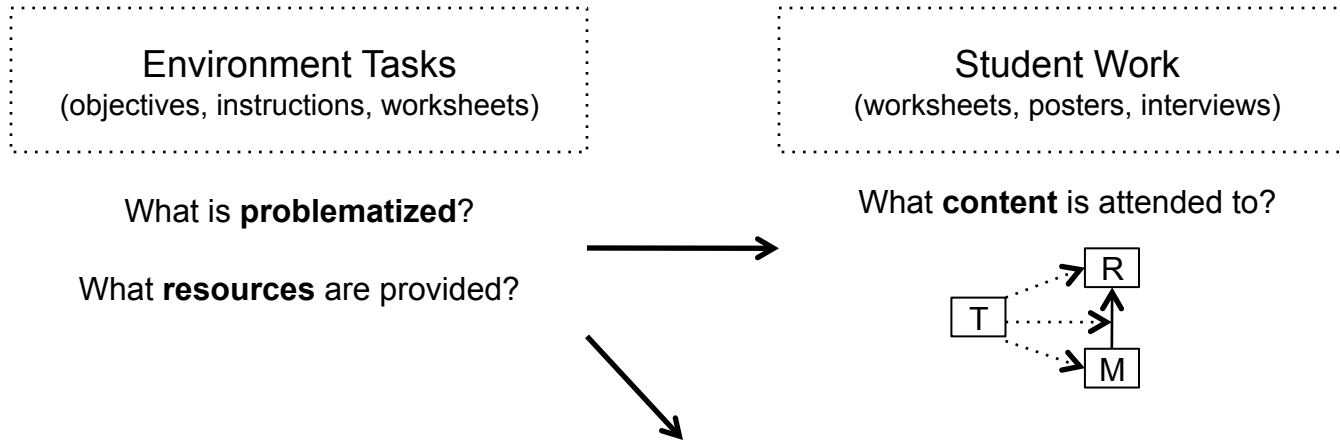
A more productive alignment would be:

- c) align directly and immediately to a **connection** between math and robots

Outline of Studies



Data Sources



Pre-Post Surveys of **Learning** and **Engagement**

Problem Solving	Math Value for Robotics	Robot Interest	Math Interest
<p>ROBOT CONTEXT A robot moved forward 6 centimeters when it was programmed to do 4 motor rotations. The programmer needed to make her robot move forward 24 centimeters. How many motor rotations does she need to enter in her program to do her move correctly?</p> <p>NON-ROBOT CONTEXT A printing press takes exactly 12 min to print 14 dictionaries. How many dictionaries can it print in 30 min?</p>	<p>4 Likert Scale Items</p> <p>Measured usefulness, value, relevance, and worth of math in robotics</p> <p>“I can think of many ways to use math in robotics”</p> <p>“Mathematics helps teach a person to think about robotics”</p>	<p>4 Likert Scale Items for each Interest Subscale</p> <p>Measured personal domain-specific interest in terms of affect, self-determination, experiencing flow, and lack of enjoyment (reverse coded)</p> <p>“I would even give up some of my spare time to learn new topics in robotics/mathematics”</p> <p>“Robotics/mathematics is dull and boring”</p>	

Scripted Inquiry – Learning Environment

Problem

- Build a Behavior → Investigation
 - Verify an equation

Dr. Turner's Hypothesis
 For every 360 degrees of wheel rotation, the robot travels one circumference of the robot's tires.

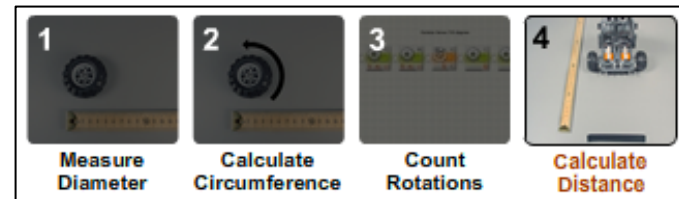
distance traveled = circumference x rotations

Investigation Goals

- Find the relationship between wheel size, motor rotations, and distance traveled by the whole robot.
- Develop a procedure that lets you convert centimeters into motor rotations so your robot can move for a distance you've measured in cm.

Resources

- Explicit step-by-step procedures



- Worksheets to record measured and calculated values

Fill in this table with the numbers you get by answering the questions in the worksheet.

Condition	Wheel Diameter (cm)	Wheel Circumference (cm)	Number of wheel rotations in program	Theoretical (predicted) distance traveled in program (cm)	Actual distance traveled (cm) in each trial	Average actual distance traveled (cm)
Standard Wheel					1.	
					2.	
					3.	
Small Wheel					1.	
					2.	
					3.	

Scripted Inquiry – Results

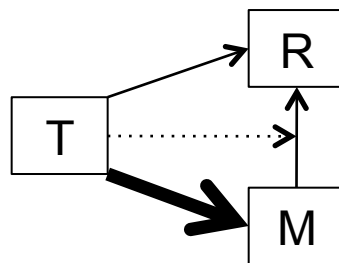
Student Work

- Math measurements, calculations, & solutions

1. Distance traveled = wheel circumference * # of rotations

2. Hypothesis & Evidence
↳ % Error

Condition	Wheel diameter	Wheel circumference	# of rotations	Theoretical	Actual	Average	% Error
Standard Wheel	5.5 cm	5.5 * π	720 degrees	17.27*	36	$\frac{36 + 35 + 37}{3}$	(34.54 - 36)
		3.14 * 5.5	720	2 or 34.54	35	2 or 36	134.54 or
		17.27	360 or 2	37	35	3	4.2%
Small Wheel	3 cm	2 * 3.14	180	18.84	19.5	$\frac{19.5 + 20 + 21}{3}$	18.84 * 20.17 = 18.84 * 100% = 7%



Outcomes

- Learning
 - ↑ Measurement & non-robots only
 - no change in Proportional reasoning or robotics
- Engagement
 - no change in Math Value for Robotics
 - no change in Robotics Interest
 - ↓ Math Interest
- Limited view of math-to-robot connections

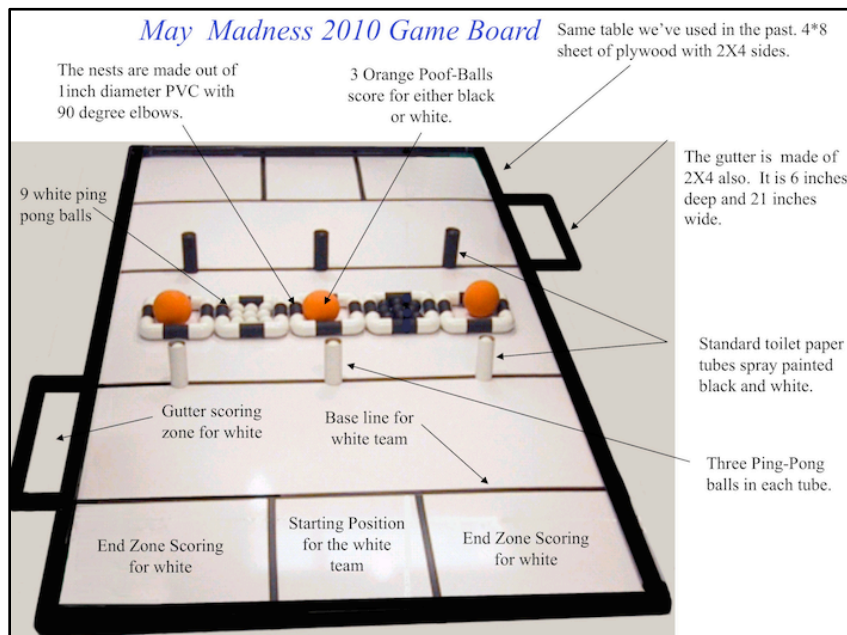
Mr. E: Do you think math is helpful for doing robots?

Darren: Yes... It has numbers. And basically what math is is numbers.

Competition – Learning Environment

Problem

- Solve a set of robot “missions”
- Get as many point as possible by collecting balls, tubes, and nests



Resources

- Each team varied considerably (nothing, books, websites)



Competition – Results

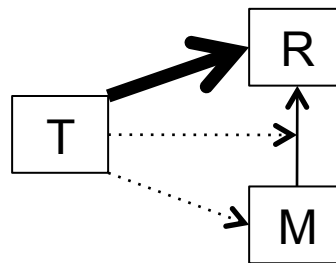
Student Work

- Program a sequence of reliable, fine-tuned movements

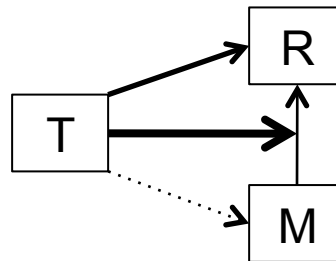


- 25% (4/16) teams used math
 - measure, calculate, adjust

- Team M1
 - Used guess-test
 - Finished 7th (out of 22)



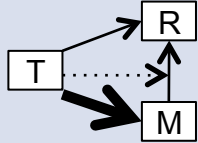


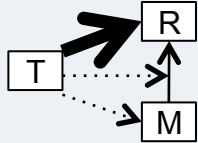
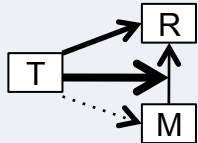

- Team M2
 - Used math strategy
 - Finished 1st



Outcomes

- Learning
 - Team M1 no change
 - Team M2 ↑
- Engagement
 - No changes on any of the subscales

- Teams each made the competition their own
 - Most reliably successful strategy for teams was entirely non-math-based

Learning Environment	Content Attended To	Problem Solving	Math Value for Robotics	Robot Interest	Math Interest
Scripted Inquiry (n = 16)		 / n.c. Measurement & non-robots only	no change	no change	
Competition M1 No math (n = 8)		no change	no change	no change	no change
Competition M2 Used math (n = 9)			no change	no change	no change

- **Aligning strongly to either math or robotics is limited**
(Scripted Inquiry & Competition M1)
- **Math as tool used directly in solving a robot problem is more productive** (Competition M2)
 - Design a learning environment focused on that connection?

Model Eliciting – Learning Environment

Problem

- Robot Synchronized Dancing (RSDv2)
 - Make a toolkit (strategy) for synchronizing
 - Model eliciting activity (Lesh et al., 2000)
 - RSDv1 focused on design of dance routine
 - Good engagement, guess-test, no learning

Resources

- Given robots and dance routine
 - Focus on synchronization
- Cycles focused on subproblems
 - Demo problem → Invent solution →
 - Share solution → Analyze teacher case

Dance Routine					Base Program
	Movement Type	Target Distance	Target Turn	Target Time	Motor Rotations
1	Straight Forward	47.1 cm		4.17 sec	5.00 rot
2	Straight Backward	75.4 cm		5.00 sec	8.00 rot
3	Point Turn Right		196 °	3.33 sec	3.00 rot
4					
5					
6					
7	Swirl				
8	Swirl				
9	Swirl				
10	Swirl				

Every time that we program the same motor rotations on both robots, Justin goes further than Beyonce by the same **relative amount**.

$\text{Move 1} \rightarrow \text{DIST}_B \div \text{DIST}_J = 47.1\text{cm} \div 88.0\text{cm} = 0.53$
 $\text{Move 2} \rightarrow \text{DIST}_B \div \text{DIST}_J = 75.4\text{cm} \div 140.7\text{cm} = 0.53$

It's always the same scale! Scale Factor = 0.53

Create a "how to" toolkit that the Bots-N-Sync captain can use to modify submitted dance routine programs so that all of the dancers do the routines in sync with each other.

Focus here on how to get robots to have **synchronized straight distances** – two the same or

Setting up the Problem

What it looks like when robots are “In-Sync”, the desired behavior



Focusing the Problem

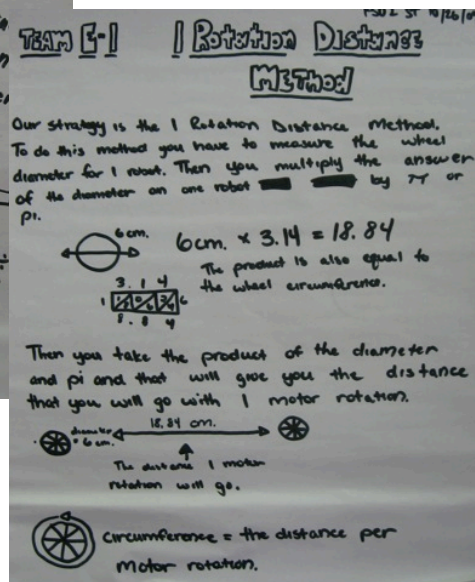
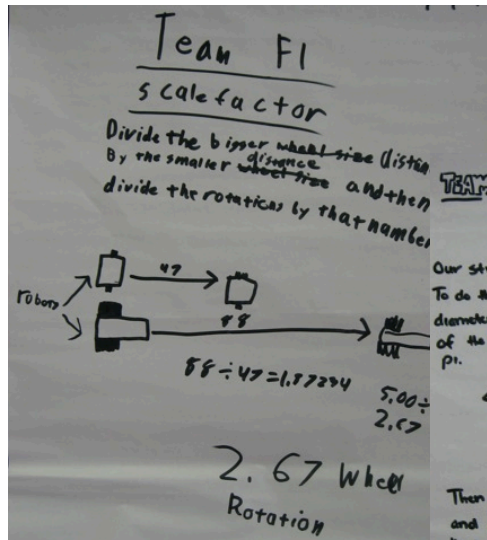
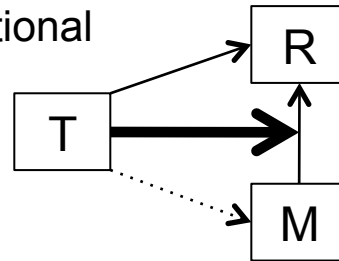
Illustrating robots “Out-of-Sync”, setting the task as adjusting programs



Model Eliciting – Results

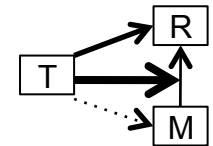
Student Work

- Developed sophisticated proportional reasoning strategies immediately
- Used scalar and functional relationships

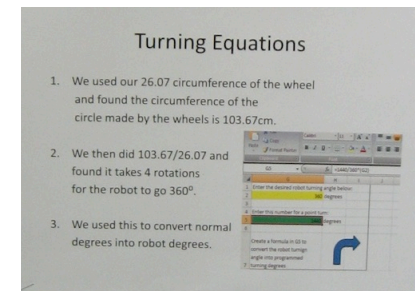


Outcomes

Competition Team (used math)

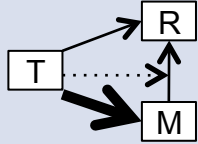


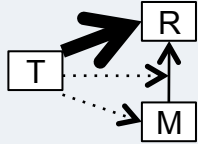
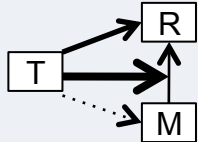

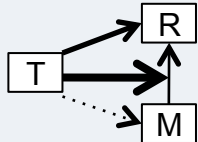
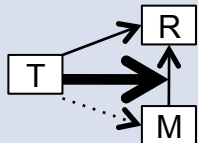





- Learning
 - no change
- Engagement
 - No changes on any of the subscales



Model Eliciting

- Learning
 - ↑ Problem Solving
- Engagement
 - ↑ Math value for robotics
 - ↓ Robot interest
 - (no change) Math Interest

Learning Environment	Content Attended To	Problem Solving	Math Value for Robotics	Robot Interest	Math Interest
Scripted Inquiry (n = 16)		 / n.c. Measurement & non-robots only	no change	no change	
Competition M1 No math (n = 8)		no change	no change	no change	no change
Competition M2 Used math (n = 9)			no change	no change	no change
Competition 2 Used math (n = 7)		no change	no change	no change	no change
Model Eliciting RSD v2 (n = 21)					no change

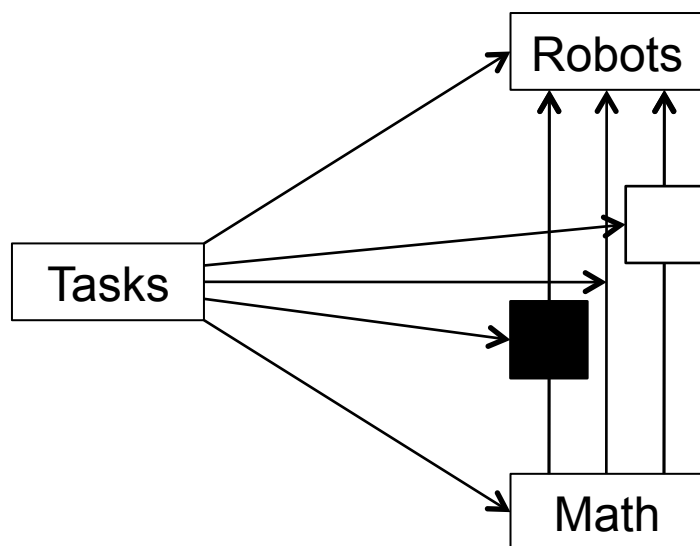
- Aligning to math-robot **connections** productive for learning (problem solving) and engagement (math value for robotics)
 - Still required hard work (robotics interest)

Part 1 – Discussion

- Summary of results
 - Differences in how learning environments problematize content & resources provided
 - Impacts problem that learners attend to
 - Adding math with robots as context nor undertaking a challenging robot problem is sufficient
 - Only **Model Eliciting** led to both ↑ problem solving and ↑ value of math for robots
 - Careful problematizing of the situation so tools are useful and providing of relevant resources
- Integrated STEM content may require careful alignment to **connections between disciplines** (Krajcik et al., 2008)
 - Learners develop meaningful, useful, sophisticated math tools for rich situations in short amounts of time
 - Keep both “in mind throughout the solution process” (Carraher & Schliemann, 2002)
 - Not shifting/translating back and forth between the situation and the math
- May be different ways to make the math-to-robot connection
 - e.g., compare math-using competition teams
 - Are some more productive for learning and engagement?
 - Investigate this possibility in **Part 2** →

Part 2 – Introduction

How do **alternative framings by learners** impact the ways in which those learners engage with and learn to use the mathematical structure of physical situations?



Rotations →  → Distance

Rotations →  → Distance

Alternative framings (Hammer et al., 2005)
students use to connect math in robots
(quasi-experimental method)



Claim that a less productive approach is to:

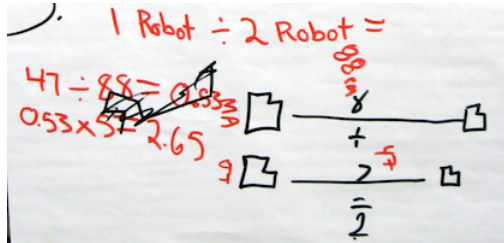
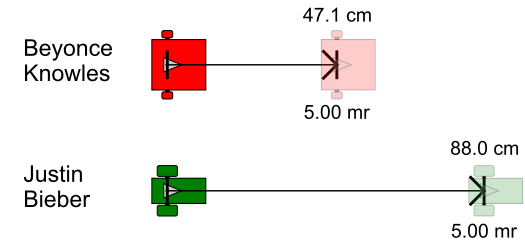
- a) use math as a tool for **transforming input values into desired outputs** (Calculational)

A more productive approach is to:

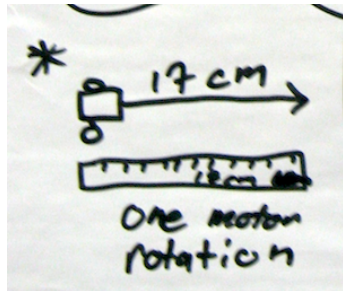
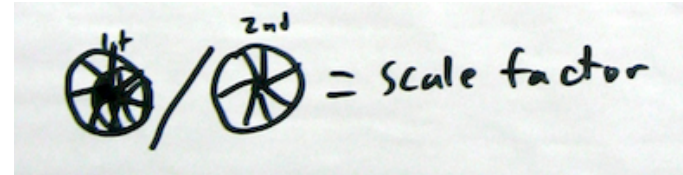
- b) use math as a tool for **representing ideas about how the robots work** (Mechanistic)

Contrasting Math-To-Robot Approaches

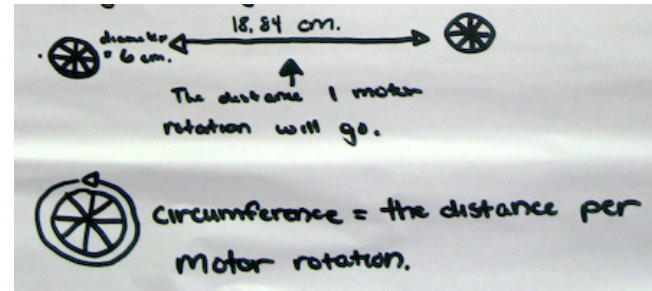
Rotations \rightarrow
 [Wheels]
 \rightarrow Distance



\neq



\neq



CALCULATIONAL

(Thompson et al., 1994)

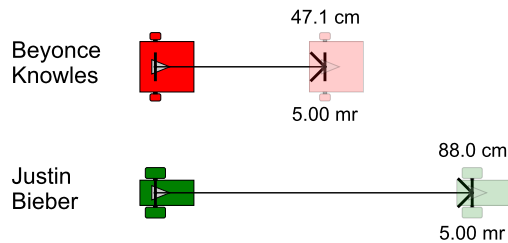
MECHANISTIC

(Russ et al., 2008)

My claim – math-to-robot approaches w/ vs w/o explicit mechanisms are **numerically** the same (use the same mathematical understanding resources), but **cognitively** different (use different physical understanding resources), so will support different learning

Study Design

Do different instructional framings of the use of mathematical resources lead to different understandings?



- Research setting 1-week in summer
- Participants – 2 Groups
 - Students assigned based on time availability, but groups randomly assigned to condition
 - 5th-7th grades (16/18 in 5th or 6th)
 - *Mechanistic* (n=10)
 - *Calculational* (n=8)
- Student Work (Posters, Discussions)
- Pre/Post Surveys from Part 1
- **Post-Instruction Competition Task**

- **Mechanistic vs Calculational (Contrasting Instructional Resources and Framings)**

- Design Task Setup

- Modeling intuitions (mechanistic) versus input-output focus (calculational)

- Teacher Cases

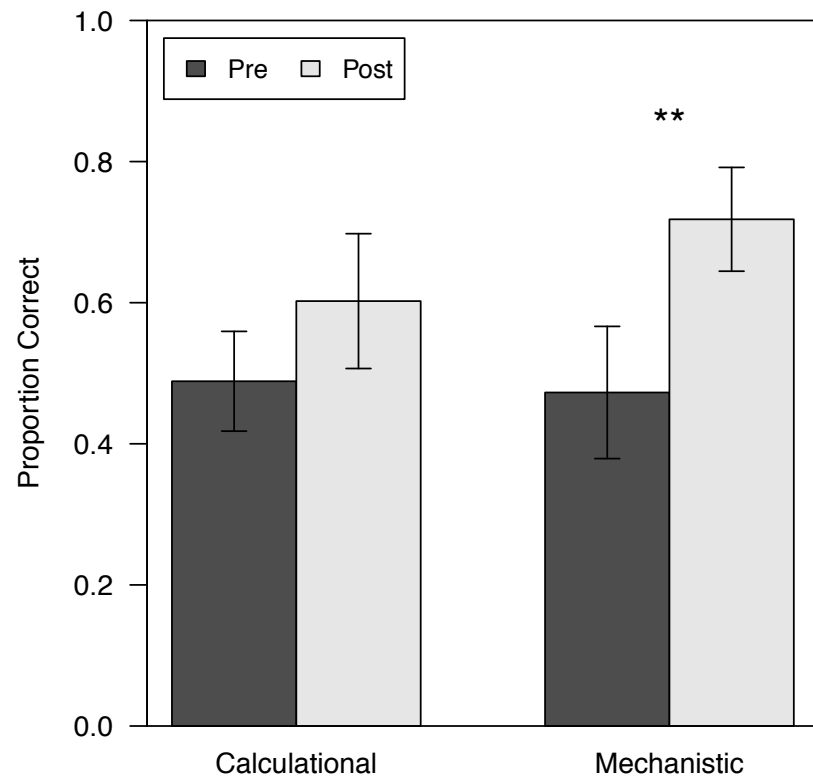
- Identifying role of physical features (mechanistic) versus identifying empirical patterns (calculational)

- Instructional Support

- Focus on explaining what quantities mean (mechanistic) versus on seeing numerical patterns in data (calculational)

Pre-Post Test Results

- Repeated Measures ANOVA suggests significant main effect of time (Pre-Post)
 - $F(1,16) = 11.05, p < .01$
- Follow-up tests suggest that only the ***Mechanistic*** Group reliably improves Pre-Post
 - Mechanistic Group
Gain = .23, 95% CI [.09, .37]
 - Computational Group
Gain = .10, 95% CI [-0.06, .26]
- **What about their work?**



Analyzing Student Work

Mechanistic

Calculational

AM
6/10

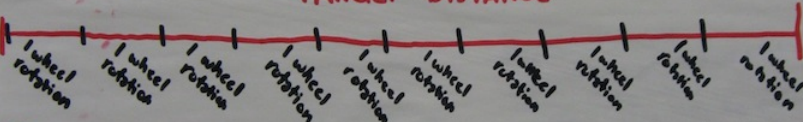
The Sectioning Method

1) Find the circumference
 Circumference = πd
 $\pi \approx 3.14$ d = diameter

2) Target Distance
 Circumference of Wheel = number of wheel rotation
 substitute the values into the equation

Explanation

TARGET DISTANCE



Sectioning Method: 1 wheel rotation represents 1 section of the target distance. The number of wheel rotations is how many sections fit into the target distance.

Exception: Your number of wheel rotations may be off by a little since you are rounding the value of π which is infinite

Example: Straight Forward #6


Target Distance: 30.2
 Circumference: $\pi \approx 3.14$ Diameter = 5.5
 $C = \pi d$
 $C \approx (3.14)5.5$
 $C \approx 17.27$

of wheel rotation: $\frac{\text{Distance}}{\text{Circumference}}$
 $\approx \frac{30.2}{17.27}$
 ≈ 1.75

PM
6/10

PURPLE NINJA

Silent But Deadly



(motor rotation) (MR) Going forward

1) $5.00 \div 2 = 2.50$

(motor rotation) (MR) (MR)

2) $2.50 + 0.246 = 2.746$

(MR) (MS)

Final Answer = 2.746, 1.20

Going Backwards

(MR) (MR)

1) $8.00 \div 2 = 4.00$

(MR)

2) $4.00 + 0.4 = 4.40$

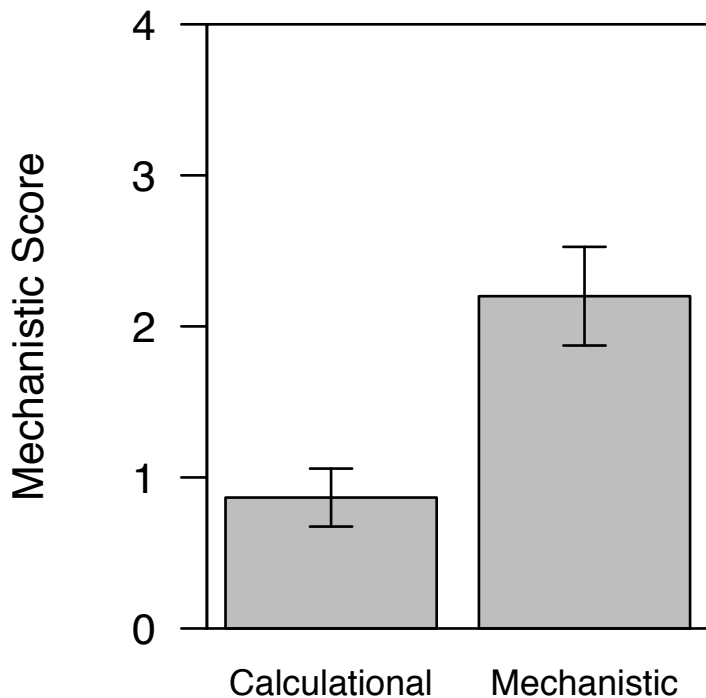
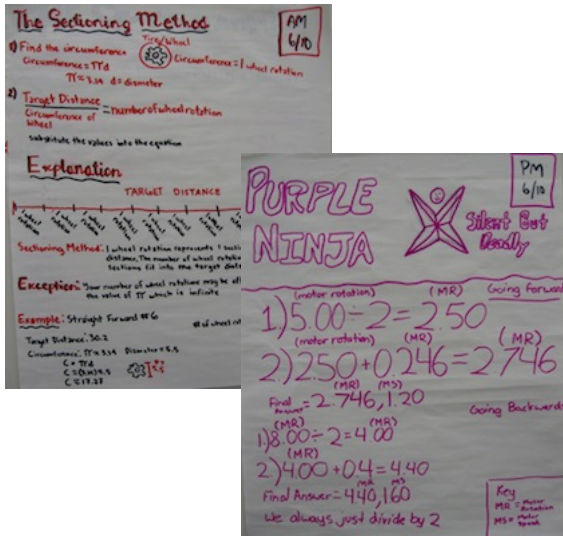
MR MS

Final Answer = 4.40, 1.60

We always just divide by 2

Key
 MR = Motor Rotation
 MS = Motor speed

Does the Mechanistic group think about the task differently? Poster Mechanistic Score

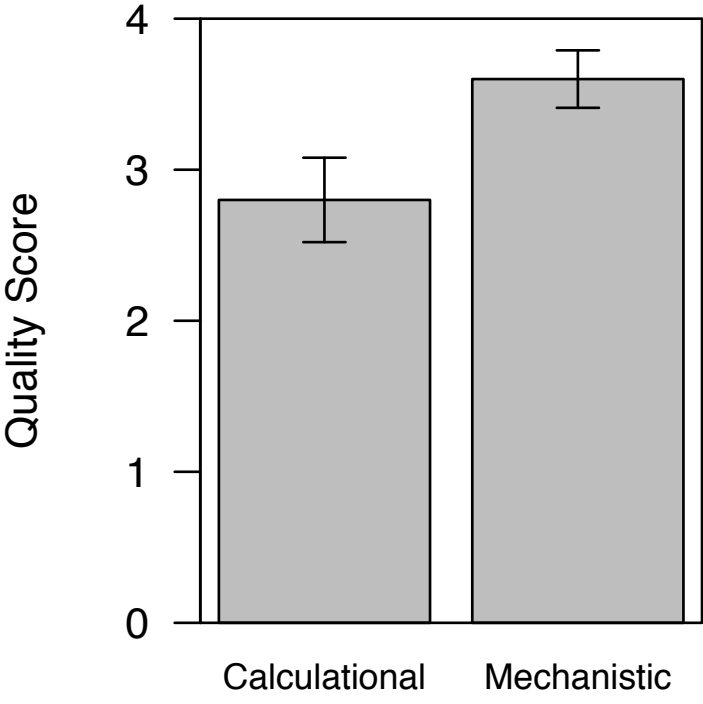
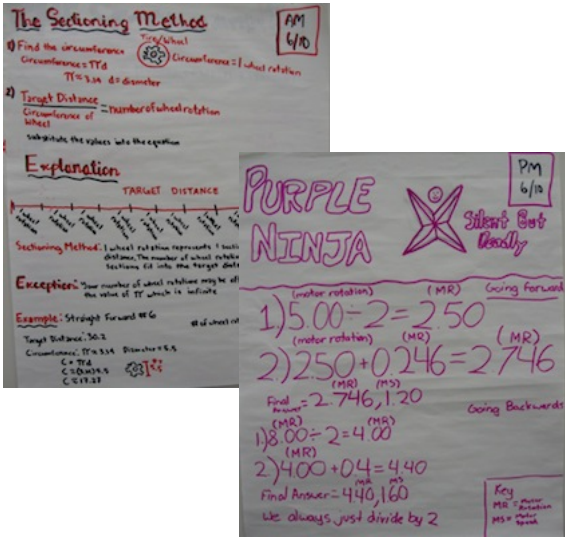


# Posters with the feature (out of 15)	Calculational	Mechanistic
Physical Features	0	6
Label Intern. Values	8	12
Situation Pictures	1	7
Explanation	4	8

- **YES**, manipulation worked well
 - Based solutions on physical features
 - Used images (not just numbers/operations)
- Mechanistic thinking **not easy**
 - **Not ALL** Mechanistic teams adopted it
 - But **No** Calculational teams did

Does the Mechanistic group invent better solutions?

Poster Quality Score



# Posters with the feature (out of 15)	Calculational	Mechanistic
Valid	13	13
Clear Steps	15	15
Fully Specified	6	15
Generalized	8	11

- **SORT OF**, no differences in some ways
 - Both invent strategies that work (valid)
 - Both articulate strategies well
- Important differences in other ways
 - Less reliance on adjusting or guessing
 - More generalizing beyond current context

Do the Computational teams just do low-level math? (procedures without connections) **NO!!**

- They do connect their math to the situation (in terms of inputs & outputs)
 - “Since Beyonce’s always half as slow as Justin, we decrease Justin’s speed by half”

Step 1 - divide Beyonce's speed by two
ex. $\frac{0.85}{2} = 0.42$
Step 2 - add 0.05
ex. $0.42 + 0.05 = 0.47$

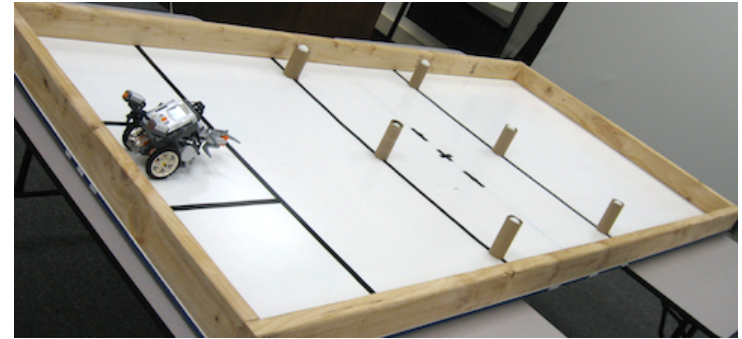
- They do make connections to and build off each other’s ideas
 - “It’s showing the, um, like how, sort of like how the Green team had divided by two, but we wanted it more exact number ... the more exact number of how much the time, of how much the speed is. It’s a bit less than half the time.”

*: Step 1) divide correct time by wrong time. Step 2) Motor rotations speed divided by quotient of step 1.
① $2.22 \sqrt{\frac{1.97}{4.17}}$ ② $1.87 \sqrt{11.20}$

- They make **math-to-robot connections** sensibly, meaningfully, but in a limited way
 - Don’t use physical features or mental animations/images to focus or evaluate their mathematical choices

Transfer Competition Task

Did you use any of the strategies from this week?



Mechanistic (4/4 teams)

- Purple Team
 - S1: We used the, the strategies that we learned all throughout the week. Um, we, like, for the straights, we, um, used the circumference of the wheel as the rotations and measured it, measured the area.
 - I: What do you mean by measured the area?
 - S2: Like how far it was from here to here. And then we like said, I think the wheel was 26 cm, so we said one rotation would be 26 cm, two would be whatever that is times two.

Mechanistic teams see the underlying similarities between the problems

Calculational teams see this as a new problem (different robot, not comparing)

Calculational (1/4 teams)

- Red Team
 - S: “Not really. No. Cause there isn’t any, like, it isn’t like we are comparing two different robots to do the same thing. All robots are the same in this ... So there really is no need for any strategies like that.”
- Purple Team
 - S1: “Cause it’s a different robot. It has bigger wheels.”
 - S2: “Well, we don’t know like, I don’t really know why we didn’t use one of our strategies. We just decided to use one and didn’t really think about the others.”
 - S1: “We’re still in the lead.”
 - I: “So it’s working for you?”
 - S1, S2: “Yeah”

Learning Environment	Content Attended To	Problem Solving	Math Value for Robotics	Robot Interest	Math Interest
Scripted Inquiry (n = 16)		/ n.c. Measurement & non-robots only	no change	no change	
Competition M1 No math (n = 8)		no change	no change	no change	no change
Competition M2 Used math (n = 9)			no change	no change	no change
Competition 2 Used math (n = 7)		no change	no change	no change	no change
Model Eliciting RSD v2 (n = 21)					no change
Calculational RSD v2 (n = 8)		no change			no change
Mechanistic RSD v3 (n = 10)			no change	no change	no change

Part 2 – Discussion

- Summary of results
 - Two groups approached the task in substantively different ways
 - Representing images/animations of mechanisms vs. being explicit about numerical patterns
 - Both engaged productively with math-to-robot problems
 - Sensibly and meaningfully connecting to the situation and building on each other's ideas
 - But the Mechanistic group
 - Improved their problem solving
 - Developed more fully specified and generalized solutions
 - Used their invented strategies in a transfer competition task
- Math-to-robot connections – math as a tool for situational understanding
 - Students have different types of **cognitive resources** available to them (Hammer et al, 2005)
 - mathematical and physical
 - The **framing** of problems make those resources more or less **accessible**
 - available and salient
 - Math resources provide set of possibilities to “organize” thinking (Schwartz et al., 2005)
 - Physical resources “focus” thinking on the most plausible organizations (Kaplan & Black, 2003)
 - They are mutually supportive and together are powerful
 - Focus more explicitly on the connection between them

Thank You

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Backup Slides

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Theory / Background

Backup Slides

Working Definitions

- Mechanistic Reasoning
 - “involves reasoning about the causes underlying physical phenomena. However, it involves more than just reasoning about causality itself—it is more than identifying the ‘X’ that causes ‘Y’ to happen. Mechanistic reasoning also requires that students think about *how* ‘X’ brings about ‘Y.’” (Russ et al., 2008)
 - “Mechanisms are entities and activities organized such that they are productive of regular changes from start or set-up to finish or termination conditions” (Machamer et al., 2000)
- Calculational Orientation
 - “driven by a fundamental image of mathematics as the application of calculations and procedures for deriving numerical results” (Thompson et al., 1994)
- Quantity and Quantitative Operation
 - Quantity - “conceptual entities ... existing in people’s conceptions of situations ... it is composed of an object, a quality of the object, an appropriate unit or dimension, and a process by which to assign a numerical value to the quality” (Thompson, 1994)
 - Quantitative Operation - “a *mental operation* by which one conceives of a new quantity in relation to one or more already-conceived quantities” (Thompson, 1994)
- (Cognitive) Resources
 - “mini-generalizations from experience whose activation depends sensitively on context.” The student compiles her explanation/conception in real time from conceptual resources that are neither right or wrong, but can be inappropriately applied. (Hammer et al., 2005)
- Productive Disciplinary Engagement
 - Engagement – students make substantive, on-task contributions coordinated with each other’s, and continually reengage over time
 - Disciplinary Engagement – contact between what the students are doing and the issues and practices of the discipline
 - Productive Disciplinary Engagement – they make intellectual progress or “get somewhere” (Engle & Conant, 2002)
- Problematizing and Problems
 - Questions, proposals, challenges, and other intellectual contributions that are considered “open” from the perspective of students, not the discipline’s perspective (Engle & Conant, 2002)
- (Instructional) Resources
 - Time, information, organizations, and other tools provided to support students in productive disciplinary engagement (Engle & Conant, 2002)
- Frames and Framing
 - “By a ‘frame’ we mean ... a set of expectations an individual has about the situation in which she finds herself that affect what she notices and how she thinks to act”
 - “we take framing as the activation of a locally coherent set of [cognitive] resources, where by ‘locally coherent’ we mean that in the moment at hand the activations are mutually consistent and reinforcing” (Hammer et al., 2005)

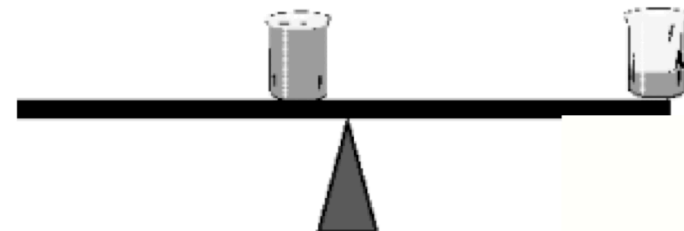
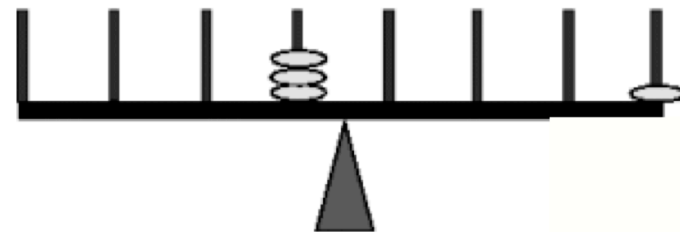
“How Mathematics Propels the Development of Physical Knowledge”

(Schwartz et al., 2005) – Which side will fall?

- Hard-to-measure quantities
(vs discrete quantities)
 - 10-yr-olds = 5yr-olds
 - Focus solely on weight (Ignore distance)
- “Show your math”
(vs “Explain your answer”)
 - 11-yr-olds = Adults
 - Use weight and distance simultaneously
- Math helps organize thinking
 - *Both quantities and operations*
 - *But limited in helping to choose between alternatives (need empirical testing)*
- Thinking about **MECHANISMS** can
(Kaplan & Black, 2003)
 - Mechanistic cues helps students engage in mental animations
 - Leads to more focused investigations of causal effects and better predictive accuracy in those investigations

Moment = Force X Distance

$$3 \times 1 \ ? \ 1 \times 4$$
$$3 < 4$$



Method / Setup

Backup Slides

Instructional Design as Method

Design/Teaching Experiments

- Understanding Goal
 - Embody conjectures about learning and instruction in designed learning environments (Sandoval, 2004)
 - Ontological Innovation (diSessa & Cobb, 2004)

Robot Environment Costs

<http://www.legoeducation.us/eng/categories/products/middle-school/lego-mindstorms-education/>

LEGO MINDSTORMS

- \$279.95
 - LEGO MINDSTORMS Education NXT Base Set
- \$79.95 (single license)
\$339.95 (site license)
 - LEGO MINDSTORMS Education NXT Software 2.1
- \$1969.96 (save \$49.00)
 - Value Pack (6 robot base sets and software site license)
- \$99.95
 - LEGO MINDSTORMS Education Resource Set

Robotics Academy

- \$274.95 (classroom license)
 - Robotics Engineering 1: Introduction to Mobile Robotics
- \$79.95 (single license)
\$399.95 (24-seat license)
 - ROBOTC Software

Other Materials

- Computers / projector
- Board
- Poster paper
- Printouts / binders
- Markers / pens / pencils
- Competition entrance fees
- Other
 - Pipe cleaners, ping pong balls, toilet paper tubes, PVC pipe nests, foam balls, etc.

Results / Analyses

Backup Slides

Comparing the Learning Environments

Environment	Problem	Resources	Student Work	
Scripted Inquiry	Verify an equation (D=WxR)	Step-by-step procedures & canonical equations	Mathematical measurements, calculations, and solutions	
Design Based	Design a set of synchronized dance movements	Data collection & organization tools	Mostly instance-based guess-test-adjust strategies with hints of proportional reasoning	
Competition	Design fine-tuned, reliable movements for each mission	Anything goes (nothing, online, books)	Varied strategies; only $\frac{1}{4}$ explicitly math-based (used measures and patterns)	
Model Eliciting	Design a toolkit (model) for synchronizing movements	Collected and organized data; Example solutions	Mostly proportional-reasoning strategies generalized across movements	

Design Based – Learning Environment

Problem

- Robot Synchronized Dancing (RSDv1)
 - Design your own dance routine, then specify it (Mehalik et al., 2008)
 - Sync it with other robots (to motivate math)

Resources

- Given set of robots and song
- Tools for specifying/measuring
 - Data sheets to record synchronization trials, then reflect on best strategy

DESIGN SPECIFICATION (CONT.)

Dance Move List
(List the sequence of moves made by your robot.)

Robot 1 Name: _____

ADJUSTED STRAIGHT DISTANCES – REFLECTION

1. What was the **best** strategy your team used for adjusting the motor rotations? Explain why that strategy worked better than the other strategies your team tried.

MOVE #	ROBOT MOVEMENT	MOTOR ROTATIONS (REV)	MOTOR SPEED (REV/SEC)	DISTANCE TRAVELED (CM)	ANGLE TURNED (°)
1					
2					
3					
4					

Robot 1 Name: Justin Timberlake (B) Robot 2 Name: Madonna (C)

MOVE #	TRIAL #	STRATEGY FOR THIS TRIAL	MOTOR ROTATIONS (ROT)	TARGET ROBOT DISTANCE (CM)	ACTUAL ROBOT DISTANCE (CM)	EVALUATION
1	1	As a first attempt, we took the motor rotations directly from the Robot 1 dance program	3.00 r	53 cm	29cm	A B C D E We were off by 24
1	2	divided the first attempt distance by number of rotations, then estimated how many more times it should go	5.00 r	53 cm	49cm	A B C D E

Design Based – Results

Student Work

- Made some cool, individualized dances (C, B)
- Began to develop proportional reasoning ideas, when working on the synchronization part
 - Most strategies involved guess-and-test

The image shows two pieces of handwritten student work for Team D. The top piece is a note explaining their use of random numbers: "We used random numbers such as 9, 7, & 5, when we tried 5 it was close but not exact so we tried 10 but it was too much so we tried everything between 9 and 10, when we tried 9.5 and got it." The bottom piece is titled "TEAM D STRATEGY" and details their approach to distance and speed. It includes calculations for distance (5.5 = 95cm, 5.75 = 100cm) and speed (.5 = 11.48 sec, 1.00 = 6 sec). A list of speed values is provided: .7 motor = 8.19 sec, .69 = 8.43, .71 = 8.19?, .72 = 7.96, .715 = 8.19. The final conclusion is "The total final answer was 0.72". To the right of the notes is a diagram with a box labeled 'T' on the left and two boxes labeled 'R' and 'M' on the right. Arrows point from 'T' to 'R' and 'M', with a dashed arrow pointing from 'M' to 'R'.

Outcomes

- Learning
 - (no change) Problem solving
- Engagement
 - ↑ Math value for robotics
 - (no change) Robot Interest
 - (no change) Math Interest
- Conclusions
 - More closely aligned to math-to-robot connections
 - Considerable effort on designing personalized dance routines
 - Focused on math-to-robot connection only in implementation second half
 - Only scratched surface of proportional reasoning
 - Timely introduction of canonical ideas/strategies?

Poster Analysis


High Mechanistic

- Mechanistic Score
 - ✓ Physical Features
 - ✓ Label Intermediate Values
 - ✓ Situation Pictures
 - ✓ Explanation
- Quality Score
 - ✓ Steps Clear
 - ✓ Valid
 - ✓ Fully-Specified
 - ✓ Generalized

AM
6/10

The Sectioning Method

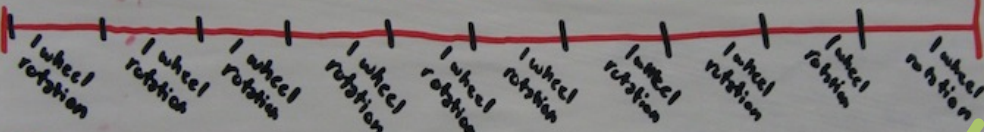
1) Find the circumference
 $Circumference = \pi d$
 $\pi \approx 3.14$ $d = \text{diameter}$

Tire/Wheel  $Circumference = 1 \text{ wheel rotation}$

2) Target Distance = number of wheel rotation
 $\frac{\text{Circumference of Wheel}}{\text{Circumference of Wheel}}$
 substitute the values into the equation

Explanation

TARGET DISTANCE



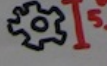
Sectioning Method: 1 wheel rotation represents 1 section of the target distance. The number of wheel rotations is how many sections fit into the target distance.

Exception: Your number of wheel rotations may be off ^{by a little} since you are rounding the value of π which is infinite

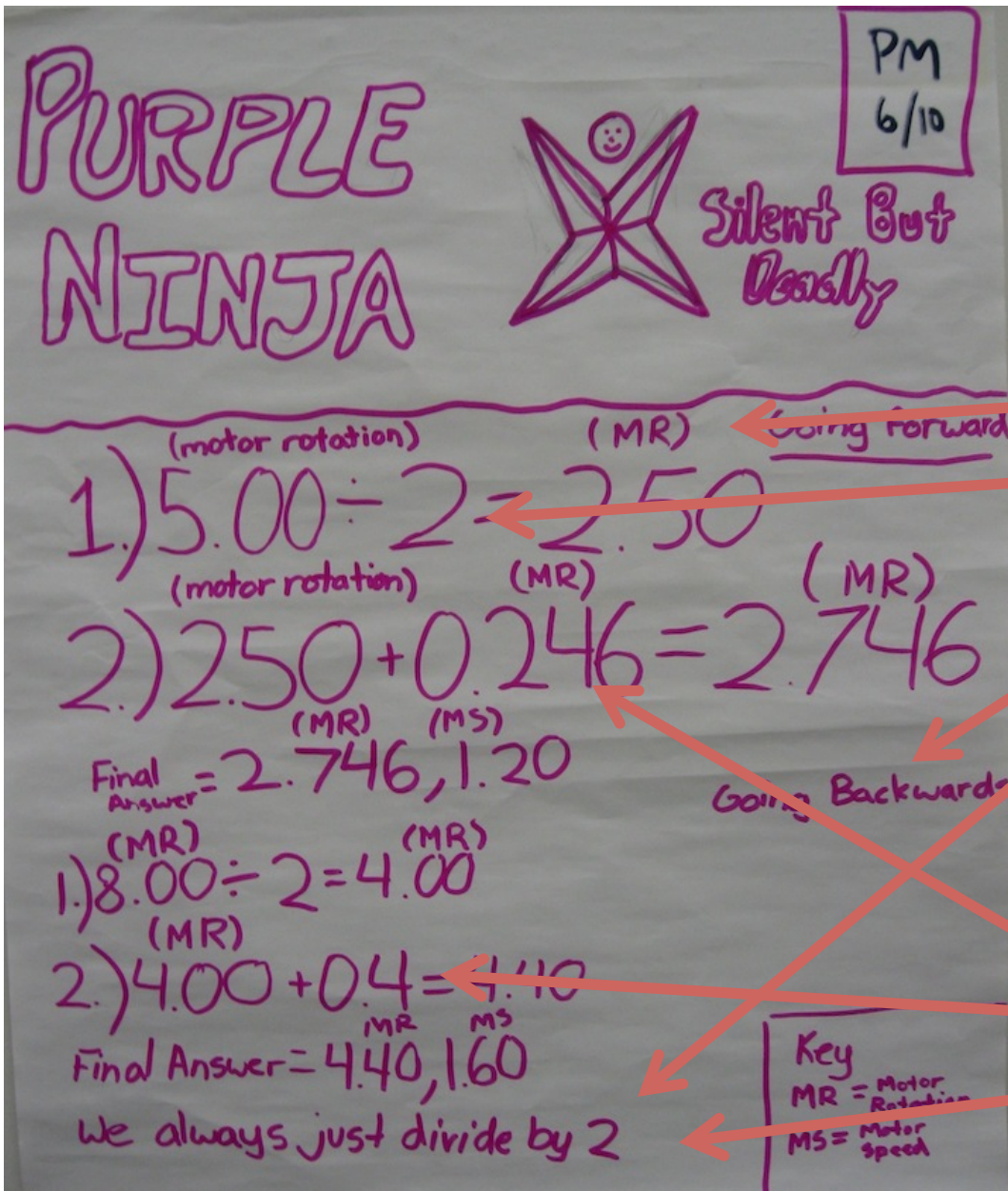
Example: Straight Forward #6

Target Distance: 30.2
 Circumference: $\pi \approx 3.14$ Diameter = 5.5

$C = \pi d$
 $C \approx (3.14)5.5$
 $C \approx 17.27$

 $d = 5.5$

of wheel rotation: $\frac{\text{Distance}}{\text{Circumference}}$
 $\approx \frac{30.2}{17.27}$
 ≈ 1.75

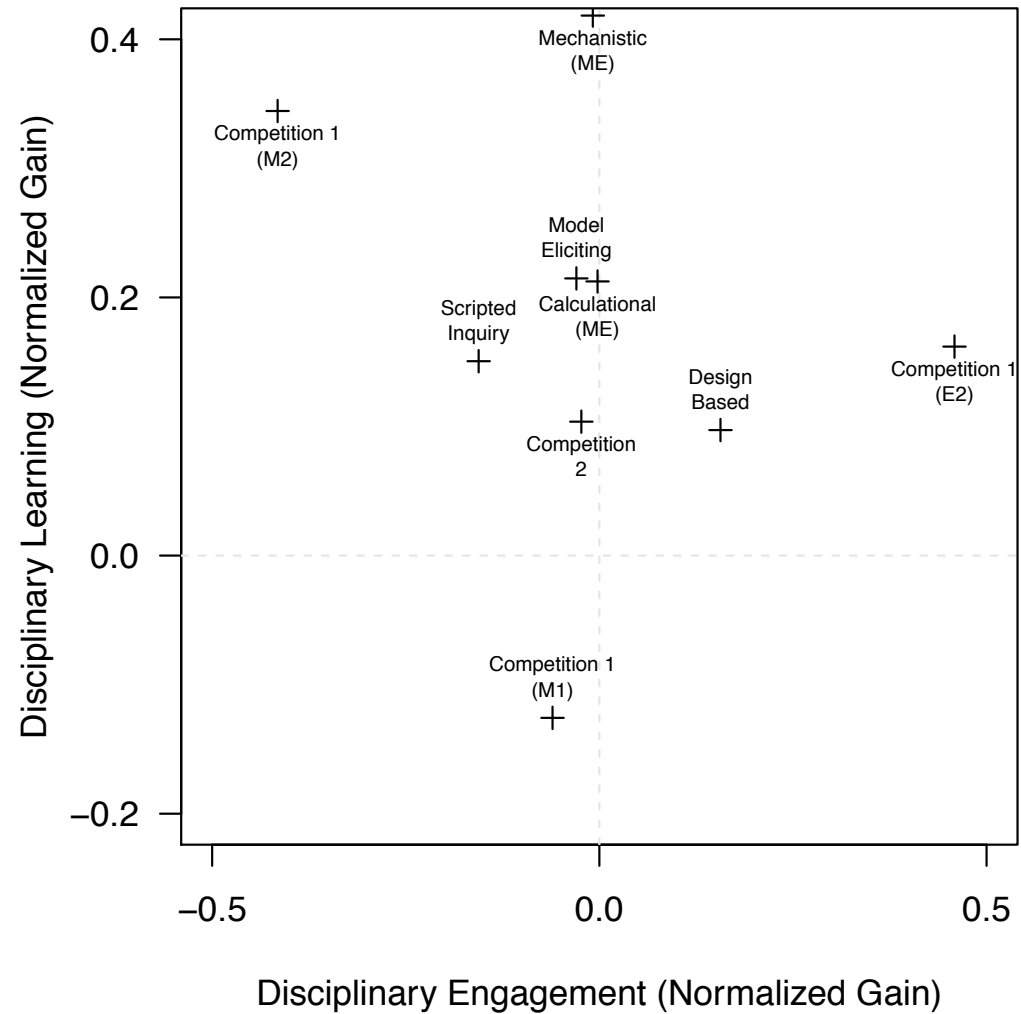
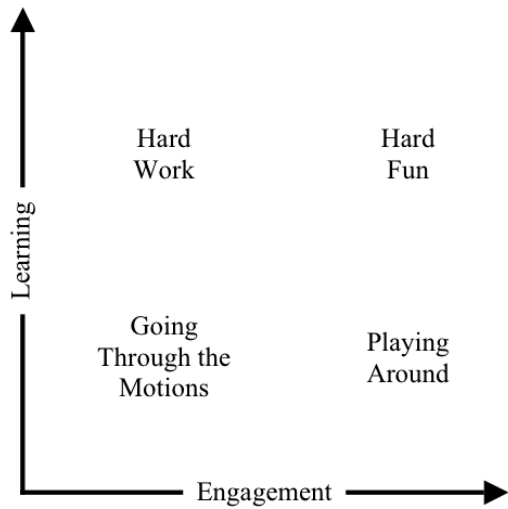


Poster Analysis

Low Mechanistic

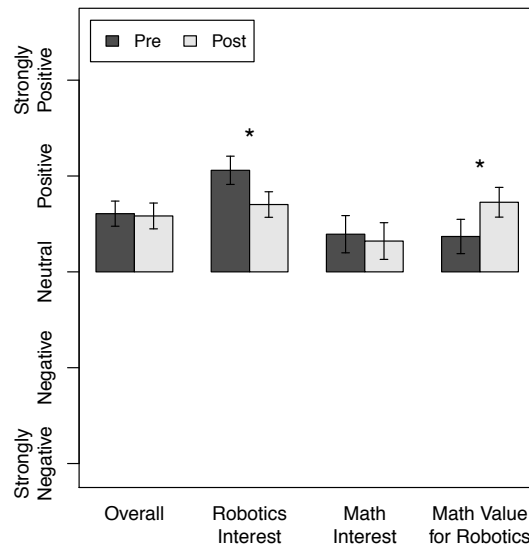
- Mechanistic Score
 - ✗ Physical Features
 - ✗ Label Intermediate Values
 - ✗ Situation Pictures
 - ✗ Explanation
- Quality Score
 - ✓ Clear Steps
 - ✓ Valid
 - ✗ Fully-Specified
 - ✗ Generalized

Space of Learning and Engagement Outcomes

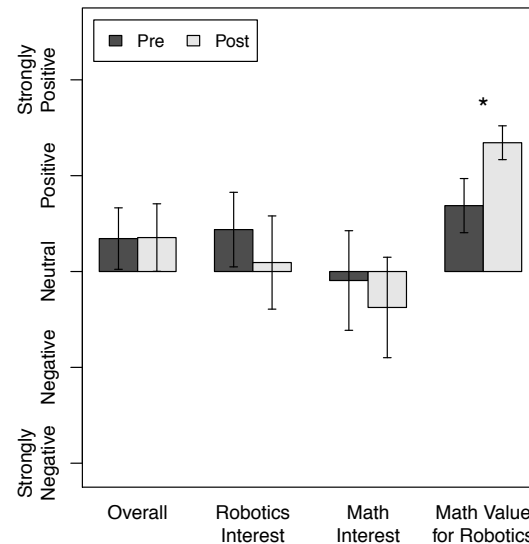


Engagement in Model Eliciting Env.

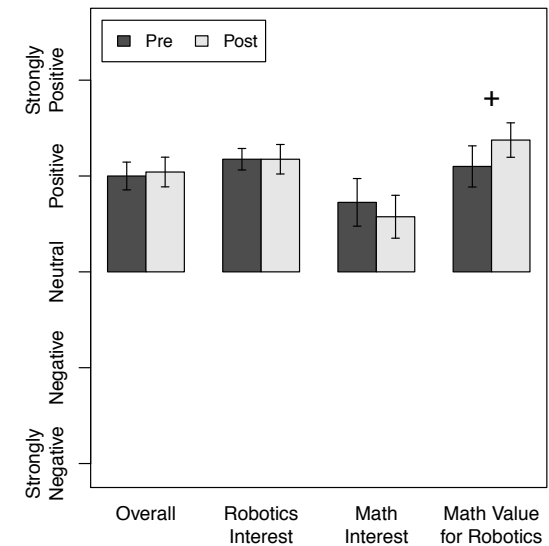
Model Eliciting (RSD v2)



Calculational (RSD v2)



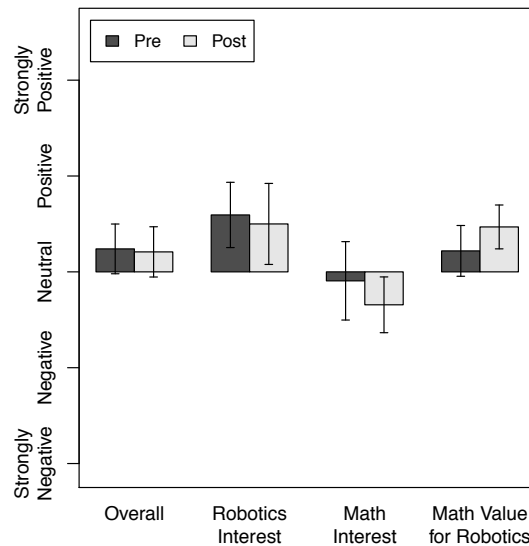
Mechanistic (RSD v3)



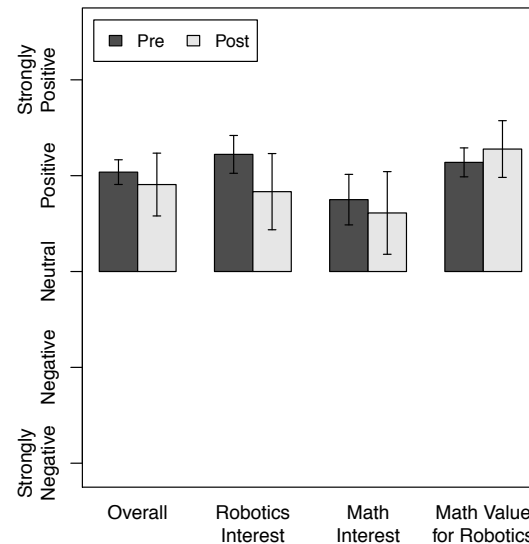
- More variability in the Calculational Group
 - Started with lower interest in Math
- Mechanistic group started with a pretty high view about the value of math for robots

Engagement in Competition Env.

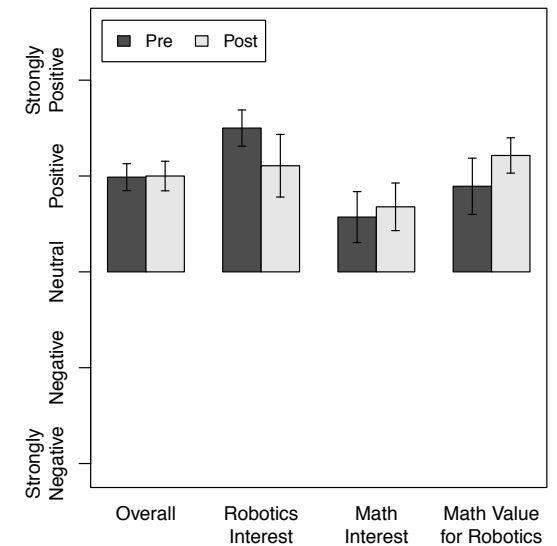
Competition M1 (no math)



Competition M2 (yes-math)



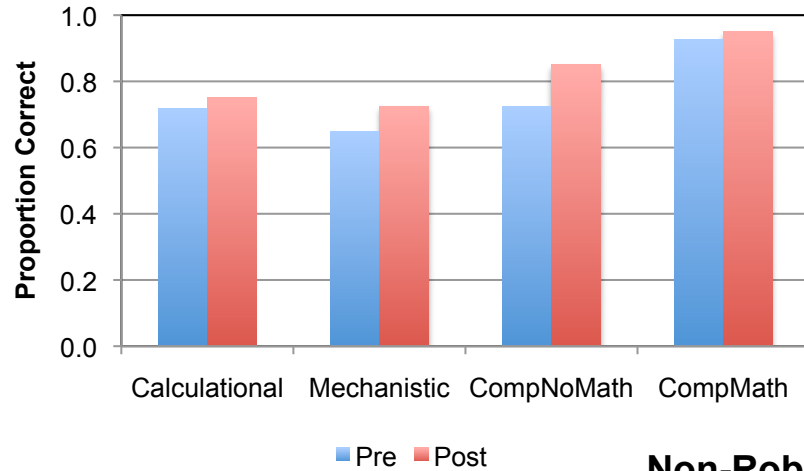
Competition 2 (yes-math)



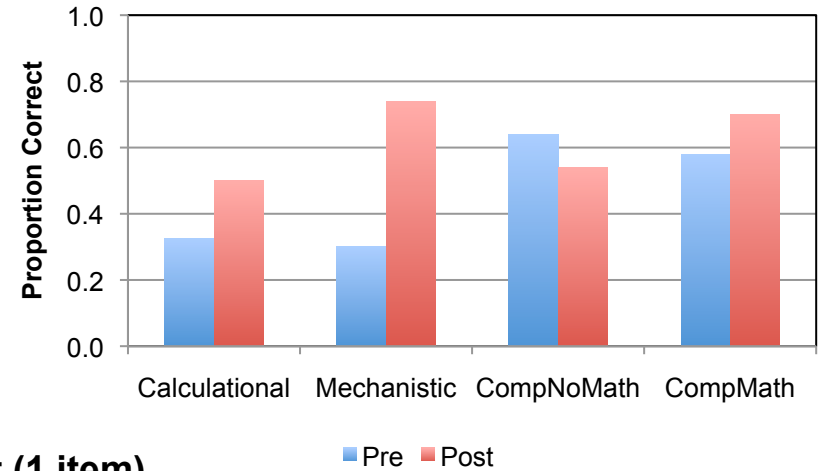
- Competition M2 and Competition 2 are high across the board at pre and post
- Competition M1 not as high

Item Type Analyses

Standard Missing Value (4 items)



Complex (5 items)



Non-Robot Context (1 item)

